Numerical Simulation in Physics and Engineering with FreeFem++

Frédéric Hecht*, Ionut Danaila†, Pierre Jolivet†, Frédéric Nataf†, Pierre-Henri Tournier†, Victorita Dolean†

FreeFem++ is a powerful and flexible software to solve numerically partial differential equations (PDE) in R^2 and in R^3 with finite elements methods. The FreeFem++ language allows for a quick specification of linear PDE's, with the variational formulation of a linear steady state problem and the user can write their own script to solve non linear problems and time dependent problems. You can solve coupled problems or problems with moving domain or eigenvalue problems, do mesh adaptation, compute error indicators, etc... This talk will give an overview of the main characteristics of FreeFem++ and demonstrates its application on examples:

1. Academic examples: Stokes, Navier-Stokes, Elasticity, ...
2. The simulation of solid-liquid phase change systems where he model are:
   - In the liquid phase, the natural convection flow is simulated by solving the incompressible Navier-Stokes equations with Boussinesq approximation
   - A variable viscosity model allows the velocity to progressively vanish in the solid phase, through a intermediate mushy region.
   - The phase change is modelled by introducing an implicit enthalpy source term in the heat equation.

   The final system of equations describing the liquid-solid system by a single domain approach is solved using a Newton iterative algo- rithm. The space discretization is based on a P2-P1 Taylor-Hood finite elements and mesh adaptivity by metric control is used to accurately track the solid-liquid interface.
3. Microwave tomography is a novel imaging modality holding great promise for medical applications and in particular for brain stroke diagnosis. We demonstrated on synthetic data the feasibility of a microwave imaging technique for the characterization and monitoring of strokes. Using high performance computing, we are able to obtain a tomographic reconstruction of the brain in less than two minutes.

   Our work demonstrates on synthetic data the feasibility of a microwave imaging technique for the characterization of CVAs, and won our research team the Bull-Joseph Fourier Prize in 2015. The numerical framework is based on high-performance computing open-source tools developed by our research team: the HPDDM library is an efficient parallel implementation of Domain Decomposition Methods (DDM) and is interfaced with the finite element software FreeFem++.

References

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https://github.com/hpddm/hpddm
http://www.freefem.org

*LJLL, UMPC, Sorbonne Universités, Paris. Email: frederic.hecht@upmc.fr
†LJLL, UMPC, Sorbonne Universités, Paris