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A Boussinesq model for thermodynamics simulations of eco-efficient courtyards

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Temperatures in courtyards are cooler in summer than temperatures outdoors, and vice versa in winter. In this work we propose a model whose goal is to simulate the temperature evolution in a courtyard with 3D parallelepiped shape, taking into account solar radiation, air temperature, air velocity and density variation.

One of the main difficulties is the computation of the surface temperature of the courtyard's walls. This is taken into account in the definition of the boundary conditions for the Boussinesq model. Actually, Robin boundary conditions are considered, which are defined in terms of the difference between the temperature of the air and the computed surface temperature.

To compute the surface temperature (T_w), it is necessary to consider a coupled system, where the second unknown is the inner temperature of the wall (T_{int}). At each point of the surface, we have a system of two differential equations depending on the density, the specific heat of the wall, the film exterior coefficient, the thickness of the wall, the transmittance coefficient of the wall, its emissivity and absorption, the solar radiation on the wall's surface, and the net radiant heat transfer. The system is described like

$$(1) \qquad \begin{cases} \rho_w c_w V_w \dfrac{\partial T_w}{\partial t} = U_{ext} A_w (T_{ext} - T_w) \\ \qquad\qquad\qquad +U_{int} A_w (T_{int} - T_w) \\ \qquad\qquad\qquad +\varepsilon_w \sigma A_w (T_{ext}^4 - T_w^4) \\ \qquad\qquad\qquad +\alpha_w A_w I_w + Q_w, \\ \rho_w c_w e_w \dfrac{\partial T_{int}}{\partial t} = U_{int}(T_w - T_{int}), \end{cases}$$

where

1. ρ_w is the density of the wall $[kg/m^3]$,
2. c_w the specific heat of the wall $[J/kgK] = [Ws/kgK]$,
3. V_w the volume of the wall $[m^3]$,
4. U_{ext} the film exterior coefficient $[W/m^2K]$,
5. A_w the area of the wall $[m^2]$,
6. T_{ext} the temperature of the air in the courtyard $[K]$,
7. U_{int} the transmittance coefficient of the wall $[W/m^2K]$,
8. σ the Stefan-Boltzmann constant $[W/m^2K^4]$,
9. ε_w the emissivity of the wall,
10. α_w the absorption of the wall,
11. I_w the solar radiation on the wall's surface $[W/m^2]$,
12. e_w the thickness of the wall $[m]$,
13. Q_w the net radiant heat transfer $[W]$.

The net heating radiation transfer Q_w is the solution of a linear system, depending on the view-factor between the areas of the grid cells that discretize the courtyard walls. These factors depend on the courtyard shape.

To sum up, we have studied a three-dimensional Boussinesq model with Robin boundary conditions defined in terms of the surface temperature of the wall. A preliminary application to a courtyard of the University of Seville will also be presented.

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