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Convergence rate of an asymptotic preserving scheme for the diffusive limit of the p-system with damping

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In this work, we present a rigourous proof of the numerical convergence for a discretization of the p-system to the diffusive limit. The p-system, as known as isentropic gas dynamics equations in Lagrangian coordinates, reads:

$$(1) \qquad \begin{cases} \partial_t \tau - \partial_x u = 0, \\ \partial_t u + \partial_x p(\tau) = -\sigma u, \end{cases} \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+,$$

where $\tau > 0$ is the specific volume of gas and $u \in \mathbb{R}$ stands for the velocity. Here $\sigma > 0$ denotes the friction parameter. Regarding the pressure law, $p(\tau) > 0$ is assumed to be smooth enough; namely $C^2(\mathbb{R}_+^*)$. Moreover, in order to enforce the system (1) to be hyperbolic, we also impose $p'(\tau) < 0$ for all $\tau > 0$.

In order to model long time and dominant friction, we proceed to a rescaling of (1) using a small parameter ε as follows:

$$(2) \qquad \begin{cases} \partial_t \tau - \partial_x u = 0, \\ \varepsilon^2 \partial_t u + \partial_x p(\tau) = -\sigma u, \end{cases} \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+.$$

While ε goes to zero, the solutions of (2) are governed by the porous media equation given by:

$$(3) \qquad \begin{cases} \partial_t \overline{\tau} = \partial_x \overline{u}, \\ \overline{u} = -\dfrac{1}{\sigma} \partial_x p(\overline{\tau}), \end{cases} \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+,$$

or equivalently

$$(4) \qquad \partial_t \overline{\tau} + \frac{1}{\sigma} \partial_{xx} p(\overline{\tau}) = 0, \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+.$$

Rencently, Lattanzio and Tzavaras [1] rigourously proved the convergence of classical solutions of (2) towards smooth solutions of porous media equation (4) and exhibited an explicit convergence rate. To access such an issue, they used the well-known relative entropy approach.

More recently, Berthon *et al* [2], considered a similar approach to prove the convergence of approximations of (2) obtained with a semi discrete scheme towards the approximations of (4) obtained with the limit discretization, while ε goes to zero. In addition, they established the same convergence rate than Lattanzio and Tzavaras.

In this presentation, we consider the fully discrete case, with a discretization of (2) in space and time. The discretization used is obtained with the semi-implicit scheme introduced by Jin, Pareschi and Toscani [3]. While this scheme is asymptotic preserving, the approximate solutions of (2) converge towards approximate solutions of (4) while ε goes to zero. By adapting relative entropy technics to the discrete framework, we show this convergence and exhibit a numerical convergence rate similar to the results from continuous and semi discrete frameworks.

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