

Time-parallel algorithm for two phase flows simulation

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In the nuclear energy domain, system codes are dedicated to the thermal-hydraulic analysis of nuclear reactors for safety studies. We are here interested in the Cathare code developed by CEA, EDF, AREVA-NP and IRSN.

Cathare solves the 6 equations two fluid model by considering a set of conservation laws (mass, momentum and energy) for each phase liquid and vapor. The discretization is based on a finite volumes method on staggered grid in space and on a fully implicit time integration method. Typical cases involve up to a million of numerical time iterations, computing the approximate solution during long physical simulation times. On the other hand, the discretization level is kept intentionally at a coarse level to be able to handle whole systems simulations.

Cathare is used in a simulator of a reactor submitted to accidental events. This platform is dedicated to train operators and prepare crisis management exercises thus requiring **real-time response of the code**. To optimize the response time, we will consider a strategy of time domain decomposition, complementing the current space domain decomposition.

This strategy will be based on the **parareal method**, introduced in [1], that provides a strategy for "parallel-in-time" computations and offers the potential for an increased level of parallelism.

The parareal in time algorithm

The semi-discretized problem that we integrate in time can be written:

$$(1) \quad \frac{\partial U}{\partial t} + A(t, U) = S(U), t \in [0, T], U(t=0) = U_0$$

We split the full interval into N sub-intervals $[0, T] = \cup_{n=0}^{N-1} [T_n, T_{n+1}]$ that will each be assigned to different processor P_n . We assume that we have two time integrators:

- $G(T_{n+1}; T_n, U(T_n))$ gives a **coarse approximation** of $U(T_{n+1})$ with $U(T_n)$ as an initial condition (low computational cost).
- $F(T_{n+1}; T_n, U(T_n))$ gives a **fine approximation** of $U(T_{n+1})$ with $U(T_n)$ as an initial condition (high computational cost).

The parareal algorithm is an iterative technique where, at each iteration k , the value $U(T_n)$ is approximated by U_n^k with an accuracy that tends to the one achieved by the fine solver when k increases. U_n^k is obtained by the recurrence relation:

$$U_{n+1}^{k+1} = G(T_{n+1}; T_n, U_n^{k+1}) + F(T_{n+1}; T_n, U_n^k) - G(T_{n+1}; T_n, U_n^k)$$

Starting from $U_{n+1}^0 = G(T_{n+1}; T_n, U_n^0)$, $n = 0, \dots, N-1$.

This method consists on giving an initial condition computed with a cheap solver to each sub-interval in sequential. Then solve **in parallel** with the fine integrator over each sub-interval. Finally correct each initial condition until convergence.

Here we apply the parareal algorithm to the resolution of an oscillating manometer. This test case is proposed in [2] for system codes to test the ability of each numerical scheme to preserve system mass and to retain the gas-liquid interface.

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References

- [1] J.-L. LIONS, Y. MADAY, G. TURINICI, Résolution par un schéma en temps "pararéel", C. R. Acad. Sci. Paris, 2001.
- [2] G.F. HEWITT, J.M. DELHAYE, N. ZUBER Multiphase science and technology, volume 6.

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