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Modelling of bedload sediment transport for weak and strong flow regimes

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In this work we deal with the sediment transport phenomena in rivers, lakes or coastal areas. It is usually divided into three types: bedload, saltation and suspension. In particular we focus on the bedload transport where particles move by sliding or rolling over the bed. This phenomena is usually described by a Saint-Venant-Exner (SVE) model consisting of a hydrodynamical component for water coupled with a morphodynamical one for the sediment. The hydrodynamical part in most cases is modeled by the Saint-Venant system. The equation that describes the morphodynamical component is the well known Exner equation that is a continuity equation where the discharge is usually empirically given. Denoting by subscript w the variables corresponding to the water and s those to the sediment, the classical SVE system reads as follows:

$$\text{(SVE)} \begin{cases} \partial_t h_w + \partial_x q_w = 0 \\ \partial_t q_w + \partial_x(\frac{q_w^2}{h_w} + \frac{1}{2} g h_w^2) + g h_w \partial_x h_s + \tau = 0 \\ \partial_t h_s + \partial_x q_s = 0 \end{cases}$$

where h and q represent the thickness and the discharge respectively and τ the shear stress at the bottom. For example the classic formula proposed by Grass [3] gives $q_s = A_g |u_w|^{m-1} u_w$ where A_g a constant depending on the grain size and the kinematic viscosity and $m \in [1, 4]$ is a positive real number.
Even if the SVE model is used in a wide range of sediment applications, it accounts with important drawbacks:

- sediment discharge q_s is defined empirically;
- the mass conservation is not ensured;
- no gravitational effects are considered;
- no energy balance is associated.

Furthermore SVE model is valid just for slow bedload sediment transport, that is, for a weak interaction between sediment and fluid.

The contribution presented in this work is twofold: the first one is to present a new SVE model that counteracts the drawbacks described above. This model is formally derived from an appropriate asymptotic approximation of NS equations, thus the gravitational forces are directly included into de system. Moreover, the energy balance is ensured since the discharge is explicitly obtained and depends on the thickness of the sediment layer h_s, see [1].
The second contribution is to introduce a bilayer Saint-Venant model that generalizes the proposed SVE system in the sense that it is valid for strong and weak bedload sediment transport. The key point falls on the definition of the friction law between water and sediment that allows us to prove that it converges to the previous SVE model in the bedload regime. We also consider an extension of the bilayer model that includes a non-hydrostatic pressure correction, [2].
Numerical simulations show that the model provides promising results and behave well in low and fast transport rate regimes.

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References

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