Automatic construction of a parametric domain for the adaptive Meccano triangulation of objects with genus-zero surfaces

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Adaptivity and automatic generation as well as parameterization of arbitrary shapes to simpler ones, more affordable and easier to work with, are desirable characteristics in a mesh. For an arbitrary genus-zero surface defined in a two-dimensional space, the Meccano method [1] provides an adaptive triangular mesh that approximates the surface boundary with a prescribed tolerance. In the automatic approach, a square parametric domain is considered, and a one-to-one mapping between the edges of the parametric square and the boundary patches in the physical domain is established. Then, an iterative procedure is performed until the approximation requirement is met. This technique includes (i) Kossaczký refinement [2] of the triangular mesh in the parametric square, (ii) projection of boundary parametric nodes to its corresponding physical location attending to the mapping, and (iii) simultaneous untangling and smoothing by optimizing the location of nodes to preserve a low distortion in the transformation from parametric to physical triangular elements.

Unfortunately, when the boundary in the parametric domain (the contour of the square) does not properly represent the boundary of the physical domain, low-quality elements can appear, and neither the refinement nor the mesh optimization can reduce the distortion of such elements. In this work, we propose an innovative strategy to obtain a suitable parametric domain to prevent this troublesome situation. The procedure can be summarized as follows:

First, we obtain a high-resolution adaptive quadtree approximation of the physical domain, where any leave intersected with the boundary is smaller than a prescribed size. Then, the leaves of this quadtree are labelled to identify connected regions, which allows the domain topology to be determined. For genus-zero in two-dimensional spaces, that implies to mark quadtree leaves as inner, outer or boundary.

In the next step, we consider a new quadtree approximation of the physical domain as the tentative parametric domain. It should be as coarse as possible to obtain a simpler shape (similar to the simplest one, i. e. the square), but accurate enough to preserve the original topology and a low distance between nodes on the physical boundary and their image on the parametric space. The shape simplicity is quantified by the number of corners on its contour, while the distance between boundary nodes is controlled by setting an upper bound for its maximum value. Once these requirements are met, a dual triangular mesh of the coarse quadtree is built and then refined using the Kossaczký method until every node in the quadtree has its counterpart in the triangular mesh. These extra edges aids to capture non-square-like shapes in the boundary, obtaining the definitive parametric domain. However, geometric and topological conditions described in [3] should be satisfied, to avoid low quality or even unresolvable degenerated elements.

The described technique automatically provides an unstructured Kossaczký mesh that defines a coarse parametric domain, suitable as meccano for the proposed mesh generator, and less prone to element degeneration than the default square parametric domain. The extension to more complex topologies and generalization to varieties and spaces of higher dimension will be explored in future works.

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References

[1] R. Montenegro, J. M. Cascón, J. M. Escobar, E. Rodríguez, and G. Montero, *An automatic strategy for adaptive tetrahedral mesh generation.* Applied Numerical Mathematics, vol. 59, no. 9, pp. 2203-2217, Sep. 2009.

[2] I. Kossaczký, *A recursive approach to local mesh refinement in two and three dimensions.* Journal of Computational and Applied Mathematics, vol. 55, no.3, pp. 275-288, Nov. 1994.

[3] E. Ruiz-Gironés, A. Oliver, G. V. Socorro-Marrero, J. M. Cascón, J. M. Escobar, R. Montenegro, and J. Sarrate, *Insertion of triangulated surfaces into a meccano tetrahedral discretization by means of mesh refinement and optimization procedures.* International Journal for Numerical Methods in Engineering, vol. 113, no. 9, pp. 1488-1506, Nov. 2017.

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