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A Well-balanced Finite Volume Scheme for Shallow Water Equations with Porosity.

Application to Modelling of Open-Channel Flow through Rigid and Emergent Vegetation

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Vegetation is known to play important role in dynamic of open-channel flow. Depending on the shape, density and spatial distribution of vegetation, water depth and flow direction might be significantly modified because the vegetation roughness is much larger than the roughness of river bed [1, 2]. Shallow Water (SW) model, obtaining by depth-integrating Navier-Stokes equations under shallowness hypothesis, can provide an accurate representation of physical processes of flow through vegetation. Nevertheless, such an *explicit* modelling is not suitable in the field, because it leads to very expensive computational cost.

It seems more appropriate to use *implicit* or *macroscopic* modelling for practical application. Traditional approaches consists in adding a drag force globally or locally into the momentum equation of SW model to enhance the determination of the local velocities. A more advanced macroscopic model, that we are interested here, is to introduce a porosity term into SW model. The porosity, ϕ, represents the fraction of the plan view area available to flow. For emergent and rigid vegetation, one can consider an isotropic and depth-independent porosity. This approach is called *single porosity* (SP) model [3] whose the mass and momentum conservation equations write

$$(1)\qquad \begin{cases} \partial_t(\phi h) + \operatorname{div}(\phi h \mathbf{u}) = 0, \\ \partial_t(\phi h \mathbf{u}) + \operatorname{div}(\phi h \mathbf{u} \otimes \mathbf{u}) + \nabla\left(\dfrac{g}{2}\phi h^2\right) \\ \qquad\qquad = \dfrac{g}{2} h^2 \nabla\phi - g\phi h \nabla b - \tau_b - \tau_d, \end{cases}$$

where h represents the depth of water and $\mathbf{u}$ denotes the depth-averaged horizontal velocity with components u and v; g is the acceleration due to gravity, b is the bed elevation, τ_b stands for the friction stress and finally τ_d expresses the depth-integrated drag due to vegetation. These last two terms are estimated by empirical quadratic laws, writing

$$(2)\qquad \tau_b = g\phi h \frac{n^2 |\mathbf{u}|\mathbf{u}}{h^{4/3}}, \quad \tau_d = \frac{1}{2}\frac{a C_d h |\mathbf{u}|\mathbf{u}}{\phi}$$

in which n is Manning's coefficient and C_d is drag coefficient. The parameter $a = \frac{1-\phi}{\pi D/4}$ is often termed as *frontal area* of vegetation of effective diameter D. Since drag force acts upon the fluid which occupies only a fraction ϕ of the total volume, the total drag is thus divided by ϕ. It is evident that when $\phi = 1$, we find again SW model.

Numerical scheme for SP model has been less studied than SW model. One can see from (1) that SP model presents an additional non-conservative source term due to spatial variation of porosity. Structure of the solution is thus mathematically more complex than SW model. We aim to study a robust scheme which inherits, on one hand, the good properties of HLLC solver, such as positivity preserving, shock capturing and easy to implement; on the other hand, the scheme captures accurately steady solutions. Therefore, we have considered a suitable simple-wave approximation of solution on which exact Riemann invariants are imposed.

We detail and analyse the construction of the simple-solver. Two test cases for illustrating the attractive behaviours of the method are presented. Finally a real application with experimental data was performed.

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