Reduced Basis Method for natural convection in a variable height cavity

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In this work we present a Boussineq VMS-Smagorinsky Reduced Basis model applied to a natural convection in a cavity. We consider that the cavity domain is parametrized by a geometrical parameter that modifies the height of the cavity.

With this Reduced Basis method we are able to compute a *real-time* solution of a parametrized PDE by using a Galerkin projection onto a low dimensional space. This low dimensional space is built by certain solutions of the FE problem, called snapshots, selected throughout a Greedy algorithm, where the construction of an *a posteriori* error estimator becomes essential.

This *a posteriori* error bound estimator is developed according with the Brezzi-Rappaz-Raviart theory, and is an extension of the one introduced in [2] for the Smagorinksy turbulence model, and the Navier-Stokes equations [3, 5].

In this Boussinesq model, both the eddy viscosity and eddy conductivity are modeled by a VMS-Smagorinsky setting [1]. These terms are highly non-linear and we need to linearize them by the Empirical Interpolation Method, that allows us to store parameter-independent matrices in the offline phase. Thanks to that, in the online phase we can compute *real-time* solutions.

Finally, we present numerical results developed in FreeFem++ (*cf.* [4]), for which we show the speed-up rate in the computation of the Boussinesq solution, for different geometry configurations of the cavity.

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