Optimal Intraregional Location of Emissions in a Transboundary Pollution Dynamic Game

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Standard dynamic models used in the literature to study different types of economic and environmental problems have considered just time dependence, ignoring the spatial variable even when studying problems with an important geographical flavour. Recently, some authors have introduced this spatial dimension in different economic contexts, and analysed optimal control problems with only one decision maker. The technical difficulties that arise when optimizing in spatio-temporal domains are the reason for the lack of abundant literature in the subject. To the best of our knowledge, there is only one recent study ([1]) taking into account the spatial variable that considers agents that behave both, dynamically and strategically. The objective of the present work is to extend this research, studying the capabilities of the same model to analyse the optimal intraregional distribution of emissions.

The model considered is a J-player non-cooperative differential game, where a planar region Ω is subdivided in subregions Ω_j, $j = 1, \dots, J$. The objective of player j is to maximize its own payoff, choosing the rate of pollutant emissions in subregion Ω_j. The focus of this paper is to characterize feedback Nash equilibria of the resulting differential game.

Let $u_j(\boldsymbol{x}, t)$ for $j = 1, \dots, J$, the emission rate of subregion Ω_j. The spatio-temporal dynamics of the stock of pollution $P(\boldsymbol{x}, t)$ is given by the parabolic partial differential equation

$$\frac{\partial P}{\partial t} = \nabla \cdot (k \nabla P) - cP + F(\boldsymbol{u}), \quad \text{in } \Omega,$$

where $\boldsymbol{u} = [u_1, \dots, u_J]^T$ is the vector of emission rates, $k = k(\boldsymbol{x})$ is a local diffusion coefficient, which is assumed to be a smooth function satisfying $k_m \leq k(\boldsymbol{x}) \leq k_M$ for all $\boldsymbol{x} \in \Omega$, where $0 < k_m \leq k_M$ are two given constants. The term $-cP$ is a natural decay of the pollutant and the source term can be written in the form

$$F(\boldsymbol{u}(\boldsymbol{x}, t)) = \sum_{j=1}^{J} F_j(u_j(\boldsymbol{x}, t)) \mathbf{1}_{\Omega_j}(\boldsymbol{x}),$$

being F_j a given family of smooth functions for $j = 1, \dots, J$, and where $\mathbf{1}_{\Omega_j}$ denotes the characteristic function of subregion Ω_j. The dynamics is completed with an initial condition and a boundary condition. Objective of player j is to choose the distributed control u_j, in order to maximize its payoff

$$J_j(u_1, \dots, u_J, P_0) = \int_0^{+\infty} \int_{\Omega_j} e^{-\rho t} G_j(u_1, \dots, u_J, P) \mathrm{d}\boldsymbol{x} \mathrm{d}t,$$

where $\rho > 0$ is a given time-discount rate, P_0 is the initial distribution of the stock of pollution and G_j are the benefits from consumption net of environmental damages.

A discrete-space model is deduced (fitting the structure used in [3]) by means of aggregated variables, maintaining the three main features of the original formulation: the model is truly dynamic, the agents behave strategically and the model incorporates the spatial aspect. Some specifications (inspired in the literature of transboundary pollution dynamic games, [2]) are introduced to characterize the feedback Nash equilibria. Finally, some numerical results illustrate the optimal intraregional distribution of emissions of the pollutant in each subregion. One of the difficulties that it is necessary to confront is the high dimensionality of the Hamilton-Jacobi-Bellman system of equations that characterizes the feedback Nash equilibria.

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