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Energy stable numerical schemes for a chemo-repulsion model with linear production term

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This work is devoted to study unconditionally energy stable numerical schemes for the following parabolic-parabolic repulsive-productive chemotaxis model (with linear production term):

$$(1)\quad \begin{cases} \partial_t u - \Delta u = \nabla\cdot(u\nabla v) \ \text{ in } \Omega,\ t>0,\\ \partial_t v - \Delta v + v = u \text{ in } \ \Omega,\ t>0,\\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial v}{\partial \mathbf{n}} = 0 \ \text{ on } \partial\Omega,\ t>0,\\ u(\mathbf{x},0)=u_0(\mathbf{x})\geq 0,\ v(\mathbf{x},0)=v_0(\mathbf{x})\geq 0 \ \text{ in } \Omega, \end{cases}$$

in a $d-$dimensional open bounded domain Ω, $d=2,3$, with boundary $\partial\Omega$. The unknowns for this model are $u(\mathbf{x},t)\geq 0$, the cell density, and $v(\mathbf{x},t)\geq 0$, the chemical concentration. The problem is well-posed ([2]), because there exist global in time nonnegative weak solutions of model (1) and, for 2D domains, there exists a unique smooth classical bounded uniformly in time solution.

By using a regularization technique ([1, 3]), we propose two fully discrete Finite Element (FE) approximations. The first one is a nonlinear approximation in the original cell and chemical variables; while the second one is a linear approximation constructed using the energy quadratization technique, in which, other auxiliar variables are introduced. In addition to proving well-posedness of our numerical schemes, we show that both unconditionally energy stable schemes satisfy the mass-conservation property which is characteristic of the chemotaxis models. As far as we know, for chemorepulsion models with linear production, there are not works studying FE schemes satisfying the property of energy-stability. Finally, we compare the behavior of these schemes throughout several numerical simulations.

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References

[1] Barrett, John W.; Blowey, James F. Finite element approximation of a nonlinear cross-diffusion population model. *Numer. Math.* 98 (2004), no. 2, 195-221.

[2] Cieslak, Tomasz; Laurençot, Philippe; Morales-Rodrigo, Cristian. Global existence and convergence to steady states in a chemorepulsion system. *Parabolic and Navier-Stokes equations*. Part 1, 105-117, Banach Center Publ., 81, Part 1, Polish Acad. Sci. Inst. Math., Warsaw, 2008.

[3] Galiano, Gonzalo and Selgas, Virginia, On a cross-diffusion segregation problem arising from a model of interacting particles. *Nonlinear Anal. Real World Appl.* 18 (2014), 34-49.

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