XVIII SPANISH-FRENCH SCHOOL JACQUES-LOUIS LIONS ABOUT NUMERICAL SIMULATION IN PHYSICS AND ENGINEERING
Las Palmas de Gran Canaria, 25-29 June 2018

A two-layer shallow flow model with two axis of integration for submarine avalanches - Nº 10

J.M. Delgado-Sánchez[1], E.D. Fernández-Nieto[1], A. Mangeney,[2] G. Narbona Reina[1]

[1]Departamento de Matemática Aplicada I, Universidad de Sevilla [2]Institut de Physique du Globe de Paris, University Paris-Diderot

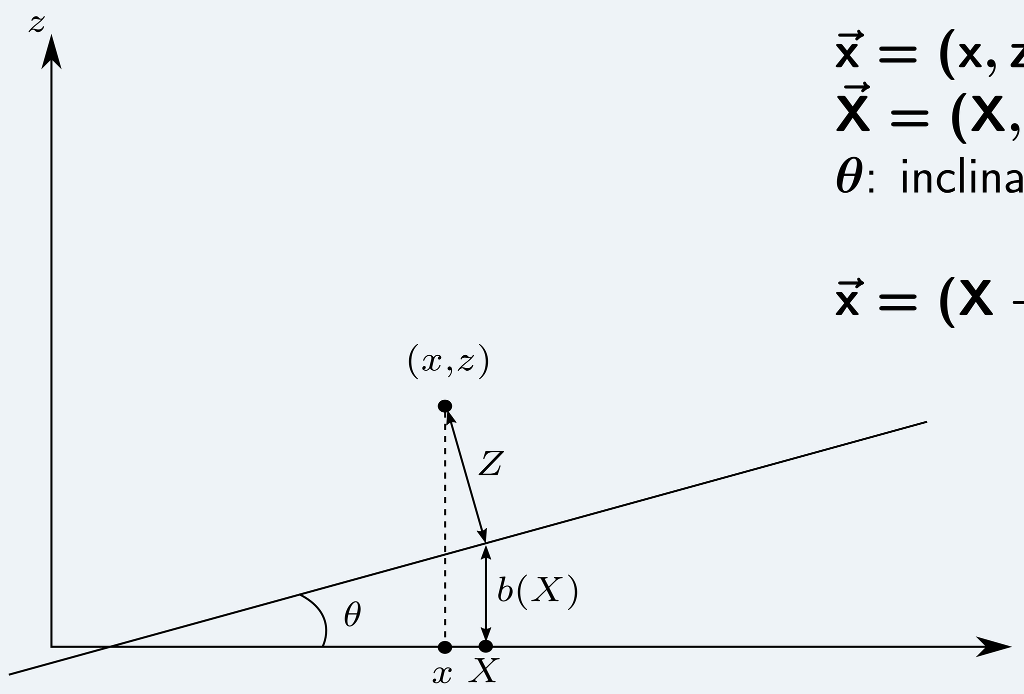
Introduction

Submarine avalanches may be described by a two-layer model of Savage-Hutter type. The modelling of both, the fluid and the granular, has usually been developed using only one coordinate system (Cartesian or local coordinates), without considering the physical requirements of the typical flow occurring in each layer.

We present a two-layer model of Savage-Hutter type to simulate submarine granular avalanches using a depth-averaging procedure of the 2D momentum and mass equations. Our approach considers two different coordinate systems: the fluid is described in Cartesian coordinates whereas local coordinates are introduced for the granular.

The proposed model

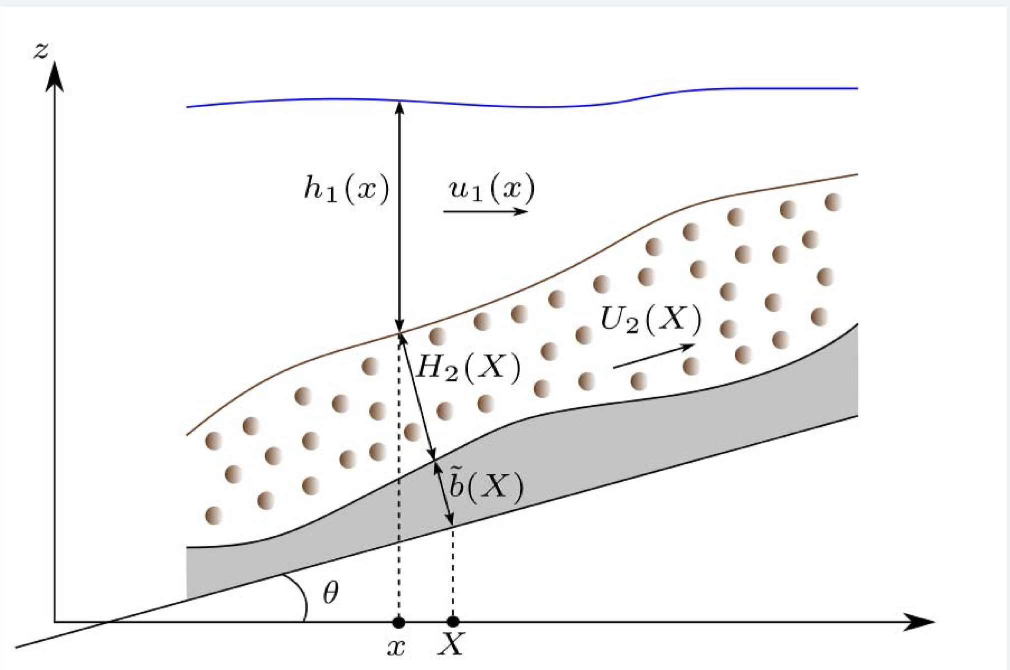
Systems of coordinates involved in the model



$\vec{x} = (x, z)$: Cartesian coordinates;
$\vec{X} = (X, Z)$: Local coordinates;
θ: inclination of the reference plane (r.p.);

$$\vec{x} = (X - Z \sin\theta, b(X) + Z\cos\theta).$$

Modelling framework



$h_1(x)$: thickness of the fluid layer;
$H_2(X)$: thickness of the granular layer (perpendicular to the r.p.);
$u_1(x)$: velocity of the fluid (horizontal);
$U_2(X)$: velocity of the grain (parallel to the r.p.);
$\tilde{b}(X)$: distance (with sign) of the bottom level to the r.p.;

Relation between the profiles sections: $x = X - (\tilde{b} + H_2)\sin\theta$.

- Governing equations: 2D incompressible Euler equations.
- Coulomb friction laws are considered at the fixed bottom.
- Friction between layers defined to obtain energy balance.

Resulting model (I)

$$\begin{cases} \widehat{\partial}_t h_1 + \partial_x(h_1 u_1) = 0; \\ \widehat{\partial}_t(h_1 u_1) + \partial_x(h_1 u_1^2) + g h_1 \partial_x(\mathcal{P}_1) = -\dfrac{F}{\rho_1}; \\ \partial_t H_2 + \cos\theta \partial_X(H_2 U_2) = 0; \\ \partial_t(H_2 U_2) + \cos\theta \partial_X(H_2 U_2^2) + g\cos\theta H_2 \partial_X(\mathcal{P}_2) = \dfrac{F}{\rho_2} - \mathcal{C} \end{cases}$$

ρ_i: densities; $r = \dfrac{\rho_1}{\rho_2} < 1$;

$\mathcal{P}_i = b + (\tilde{b} + H_2)\cos\theta + \dfrac{\rho_2}{\rho_i} r h_1$;

F: friction term between layers;

$\mathcal{C} = g \cdot \mathrm{sgn}(U_2) H_2 \cos\theta (1 - r)\tan\delta_0$ (δ_0: Coulomb friction angle).

Resulting model (II)

Change of variables for equations of layer 1 (fluid)

$$(X, t) \longmapsto (x, t) = \left(X - (\tilde{b}(X) + H_2(X))\sin\theta, t\right)$$
$$\mathcal{J} := \det(\nabla_{(X,t)}(x, t)) = 1 - \partial_X(\tilde{b} + H_2)\sin\theta;$$
$$H_1(X) := h_1(x); \quad U_1(X) := u_1(x).$$

$$\begin{cases} \partial_t(H_1\mathcal{J}) + \partial_X\left(H_1(U_1 + \partial_t H_2 \sin\theta)\right) = 0; \\ \partial_t(H_1 U_1 \mathcal{J}) + \partial_X\left(H_1 U_1(U_1 + \partial_t H_2 \sin\theta)\right) + g H_1 \partial_X(\mathcal{P}_1) = -\mathcal{J}\dfrac{F}{\rho_1}; \\ \partial_t H_2 + \cos\theta \partial_X(H_2 U_2) = 0; \\ \partial_t(H_2 U_2) + \cos\theta \partial_X(H_2 U_2^2) + g\cos\theta H_2 \partial_X(\mathcal{P}_2) = \dfrac{F}{\rho_2} - \mathcal{C} \end{cases}$$

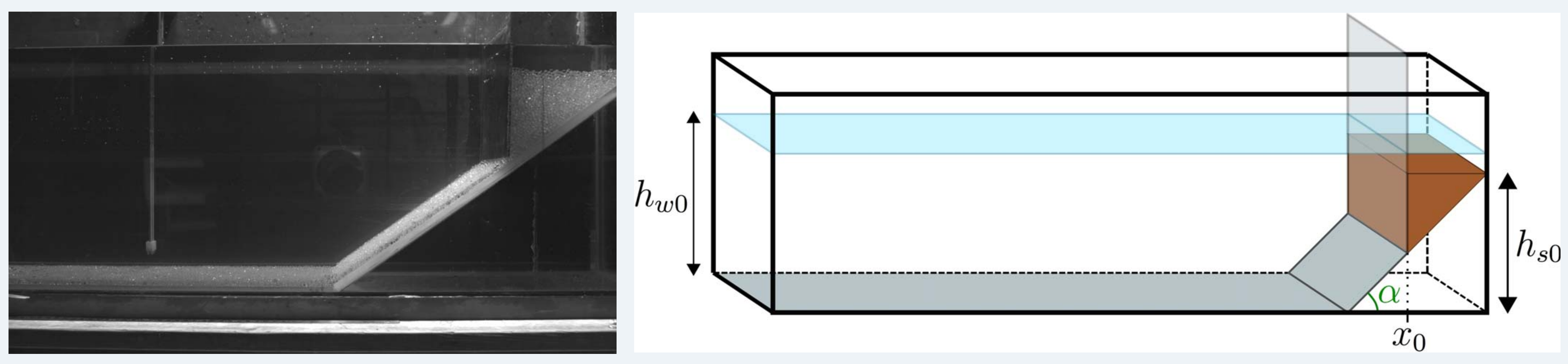
$\mathcal{P}_i = b + (\tilde{b} + H_2)\cos\theta + \dfrac{\rho_2}{\rho_i} r H_1$;

$F := \rho_1 \lambda(\mathcal{J} U_1 \cos\theta - U_2)$ $\Rightarrow$ Model with dissipative energy balance

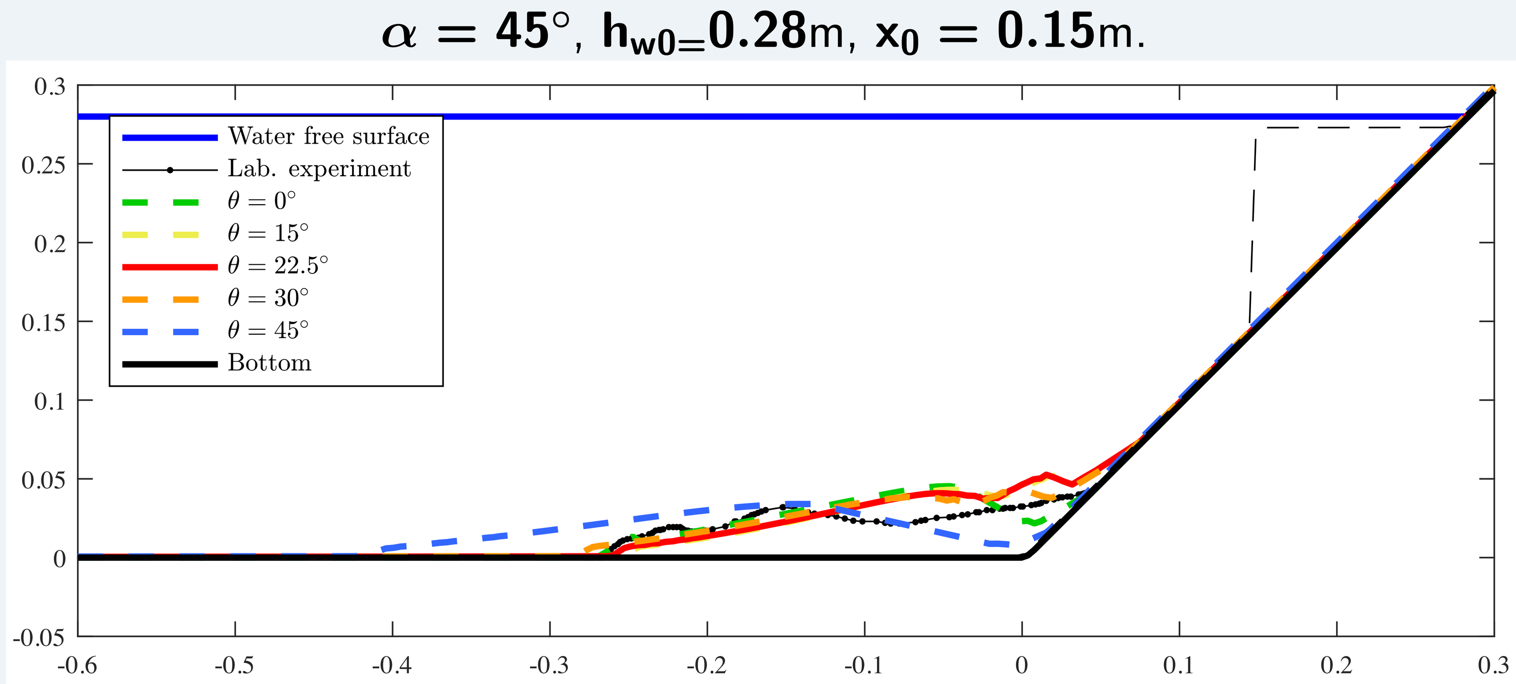
Numerical results

TEST 1: Comparison with experimental data

Goal: to show the influence of θ in the numerical solution (steady state).



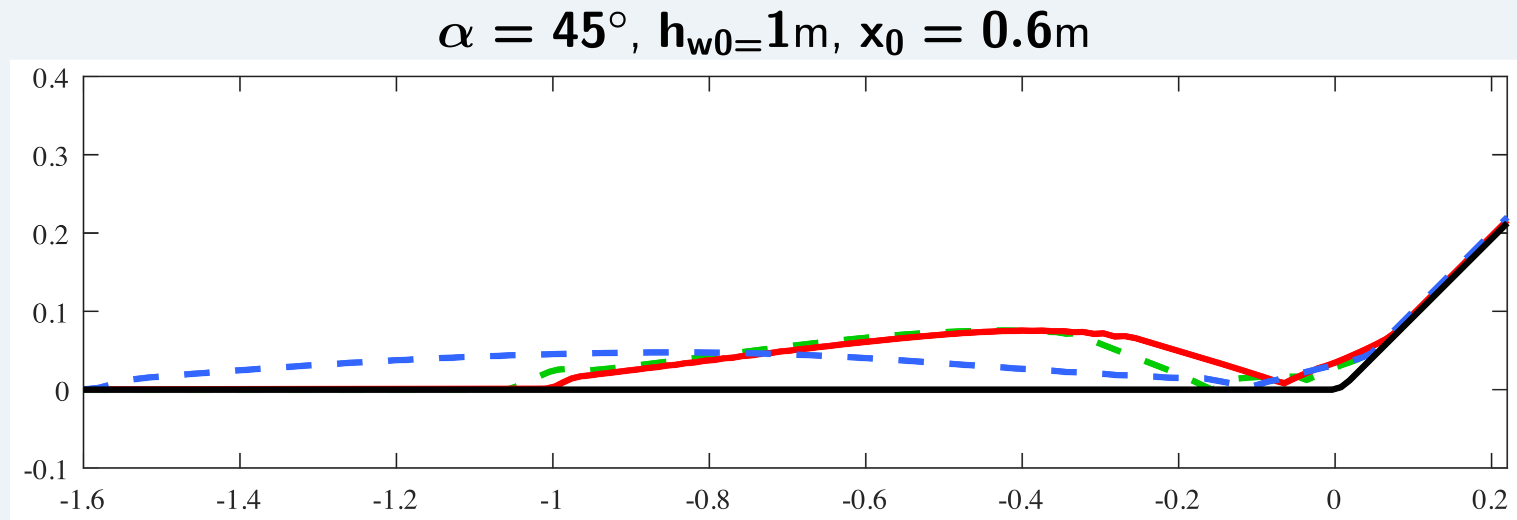
$\alpha = 45°$, $h_{w0} = 0.28$m, $x_0 = 0.15$m.



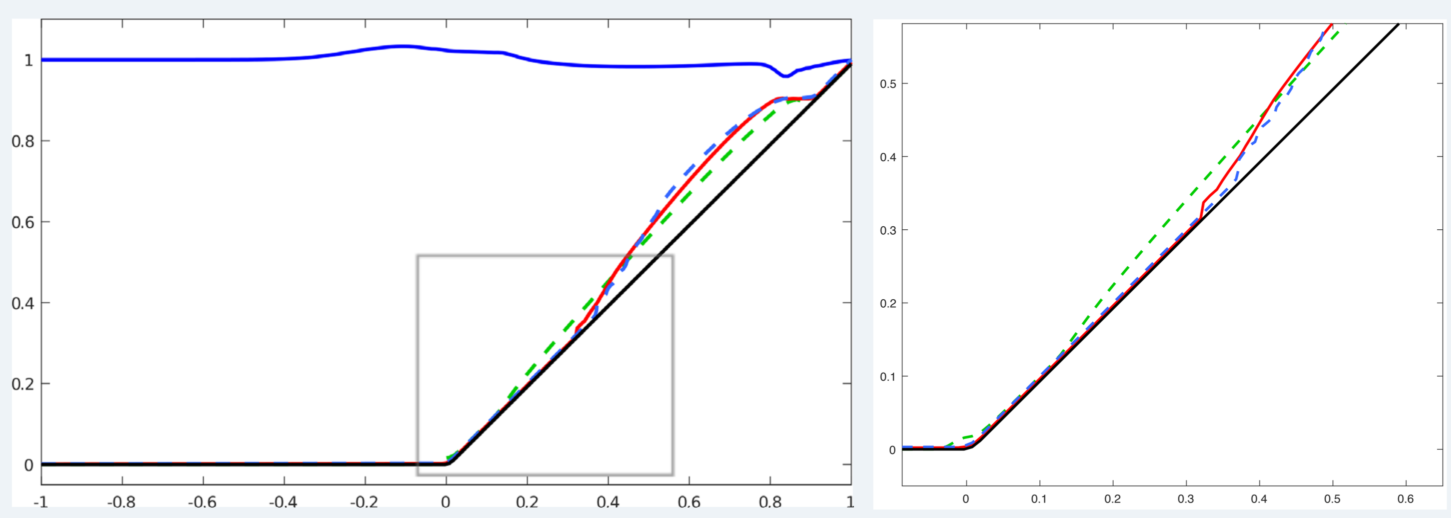
TEST 2: Influence of the slope in the reference plane

Goal: to show the influence of θ in the numerical solution (evolutive state).

$\alpha = 45°$, $h_{w0} = 1$m, $x_0 = 0.6$m



Steady state



$t = 0.3$s

Conclusions

- Remarkable influence of the slope θ of the r.p. in the numerical solutions.
- The best choice: to consider the model above with an intermediate value of θ between $0°$ and the maximum inclination of the bottom.

Acknowledgements

This research was supported by the Ministerio de Economía, Industria y Competitividad under grant MTM2015-70490-C2-2-R with the participation of FEDER.

Contact information

- Email: jmdelga@us.es