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A Boussinesq model for thermodynamics simulations of eco-efficient courtyards - N[14]

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Introduction

Nowadays, temperatures in courtyards are cooler in summer than temperatures outdoors, and vice versa in winter. In architecture, this thermodynamic phenomenon is an important issue because this advantage can be used by the refrigeration of a building. In this poster contribution we propose a model whose goal is to simulate the temperature evolution in a courtyard with 3D parallelepiped shape, taking into account solar radiation, air temperature, air velocity and density variation. It is considered to simulate the courtyard of the Institute of Mathematics of the University of Seville (IMUS). However, the model introduced is for a simplified courtyard.

In fact, the study is developed with the IMUS courtyard for all the walls with the same dimensions, without glasses and obstacles inside. Therefore, one could expect that qualitatively the behaviour is similar to Imus courtyard.

Figure : IMUS courtyard.

Courtyard temperatures representation

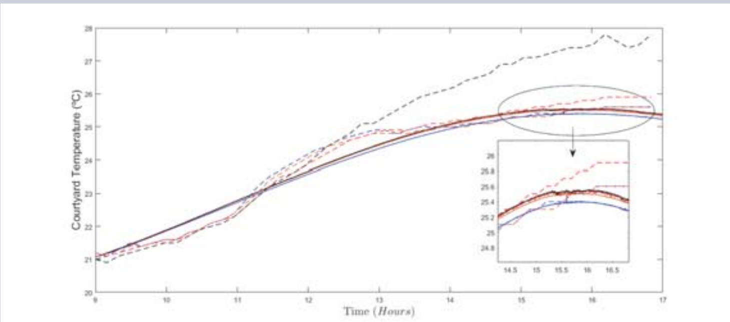

Figure : Comparison between the average of the **temperatures inside the courtyard** near of the west, east and south walls at different heights: 8m (**black**), 5m (red), 2m (orange), 1m (blue) (Continuous lines) and the **eperimental measured temperatures** of the IMUS courtyard within the same way (Discontinuous lines).

Evolution of the temperature in a courtyard

The model can be simulated by a Navier-Stokes problem with variable density depending on the temperature (Boussinesq model) as follows

$$\begin{cases} \dfrac{\partial \bar{u}}{\partial t} + \bar{u}\cdot\nabla\bar{u} - \nabla\cdot(\nu\nabla\bar{u}) + \nabla p = f \text{ in } \Omega, \\ \nabla\cdot\bar{u} = 0 \text{ in } \Omega, \\ \dfrac{\partial T}{\partial t} + \bar{u}\cdot\nabla T - \nabla\cdot(\kappa\nabla T) = \varphi \text{ in } \Omega, \\ T = T_w \text{ on } \Gamma, \\ T(0) = T_{op} \text{ in } \Omega, \end{cases}$$

where $\bar{u}$, velocity field; T, temperature inside the courtyard; t, time; ν, kinematic viscosity; p, pressure; κ, thermal conductivity; φ, source term due to the occupancy; f, density of the heating source due to the radiation with

$$f = \begin{pmatrix} 0 \\ 0 \\ g - \alpha g(T - T_{ref}). \end{pmatrix}$$

Here, g, gravitational acceleration; α thermal expansion coefficient; T_{ref}, temperature of reference in the environment.

Computation of surface and inner temperatures

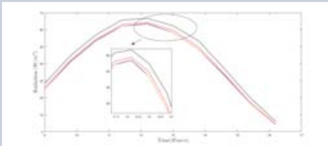

Figure : Average of the **radiation** counting the west, east and south walls at different heights: 8m (**black**), 5m (red), 2m (orange), 1m (blue).

One of the main difficulties is the computation of the surface temperature of the courtyard's walls. This is taken into account in the definition of the boundary conditions for the Boussinesq model. Actually, Dirichlet boundary conditions are considered, which are defined in terms of the surface temperature. The model introduced can be described like

$$\begin{cases} \rho_w c_w V_w \dfrac{\partial T_w}{\partial t} = U_{ext} A (T_{ext} - T_w) \\ \qquad + U_{int} A (T_{int} - T_w) \\ \qquad + \varepsilon_w \sigma A (T_{ext}^4 - T_w^4) \\ \qquad + \alpha_w A I + Q_w, \\ \rho_w c_w e_w \dfrac{\partial T_{int}}{\partial t} = U_{int} T_w \\ \qquad - U_{int} T_{int}, \end{cases}$$

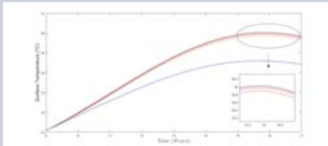

Figure : Average of the **surface temperatures** counting west, east and south walls at different heights: 8m (**black**), 5m (red), 2m (orange), 1m (blue). The net radiant heat transfer is not involved in this graph.

where T_w, surface temperature; T_{int}, inner temperature of the wall; ρ_w, density; c_w, specific heat; V_w, volume of the wall; U_{ext}, film exterior coefficient; A_w, area of the wall; T_{ext}, temperature of the air in the courtyard; U_{int}, transmittance coefficient of the wall; σ, Stefan-Boltzmann constant; ε_w, emissivity; α_w, absorption; I_w, solar radiation on the wall's surface; e_w, thickness of the wall; Q_w, net radiant heat transfer.

Conclusions

- A three-dimensional model is developed to optimize the temperature of a courtyard.
- A generalized boundary condition is used to model the surface temperature of the courtyard.
- A new quadratic formula is calculated to approximate view-factors, which are used to obtain the net radiant heat transfer, for perpendicular and parallel surfaces.

Forthcoming Research

- We are studying the inserction of the net radiant heat transfer to calculate the surface temperature.
- We are developing a theoretical study about the model.
- We aim to apply a reduced order method to decrease the cost of computation.

References

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