Time-parallel algorithm for two phase flows simulation - Nº 18

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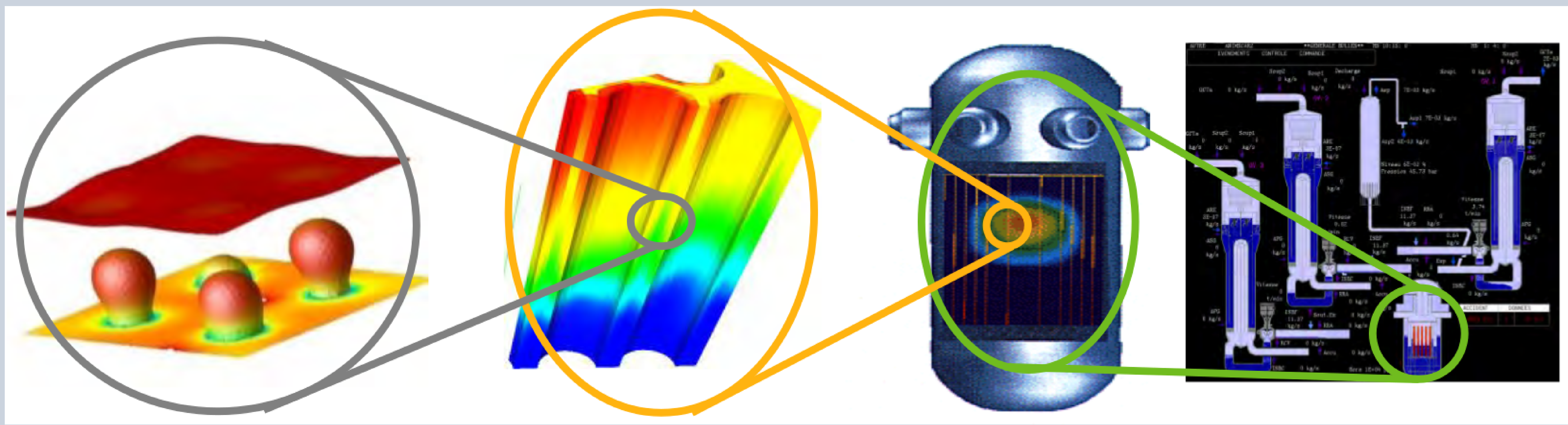
Context



In the nuclear energy domain, system codes are dedicated to the thermal-hydraulic analysis of nuclear reactors, mainly for safety and accidental situations studies. We are here interested in the Cathare code developed by CEA, EDF, AREVA-NP and IRSN. To improve the response time, we consider a strategy, complementing the space domain decomposition, based on the parareal method [LionsMadayTurinici,2001].

Cathare model

The 6 equations two fluid model. Main unknowns: $(\mathbf{p}, \alpha_v, \mathbf{u_k}, \mathbf{H_k})$

$$\begin{cases} \partial_t(\alpha_k\rho_k) + \partial_x(\alpha_k\rho_k u_k) = 0 \\ \alpha_k\rho_k\partial_t u_k + \alpha_k\rho_k u_k\partial_x u_k + \alpha_k\partial_x p = \alpha_k\rho_k g + F_k^{int} \\ \partial_t\left[\alpha_k\rho_k\left(H_k + \frac{u_k^2}{2}\right)\right] + \partial_x\left[\alpha_k\rho_k u_k\left(H_k + \frac{u_k^2}{2}\right)\right] = \alpha_k\partial_t p + \alpha_k\rho_k u_k g \end{cases} \quad (1)$$

with $\alpha_k \in [0,1]$, $\alpha_g + \alpha_l = 1$ and the two equations of state : $\rho_k = \rho_k(p, H_k)$.
Interfacial forces F_k^{int} are of 2 types:

- ensures hyperbolicity of the system [Ndjinga,2007]
- interfacial friction term to deal numerically with vanishing phase

Cathare scheme

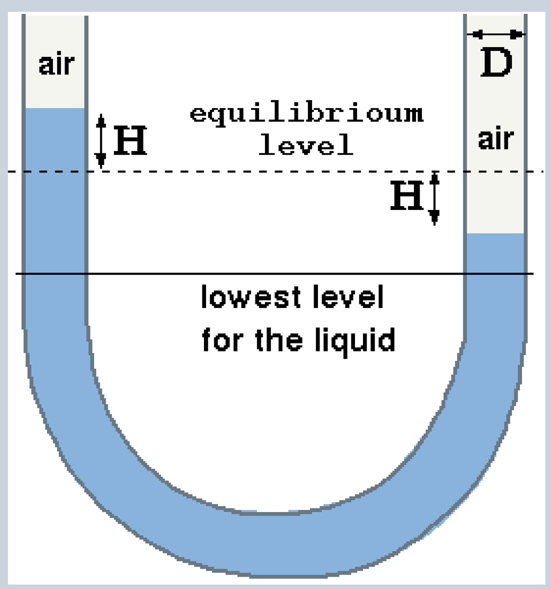


Figure : Oscillating manometer [HewittDelhayeZuber,1991]

- Initial condition: $\mathbf{P} = \mathbf{10^5}$, $\mathbf{h_l = 4.17 \times 10^5}$, $\mathbf{h_v = 2.68 \times 10^6}$, $\mathbf{u_v = u_l = -2.1}$ and $\alpha_v = \begin{cases} 1 - 10^{-5}, \text{ in the upper half} \\ 10^{-5}, \text{ elsewhere} \end{cases}$
- Time interval : [0,20]
- Error norm: $\frac{\max_n ||U^n - U^n_{ref}||_{L^2}}{\max_n ||U^n_{ref}||_{L^2}}$, $U^n = (P^n, \alpha_v^n, h_v^n, h_l^n, u_v^n, u_l^n)$

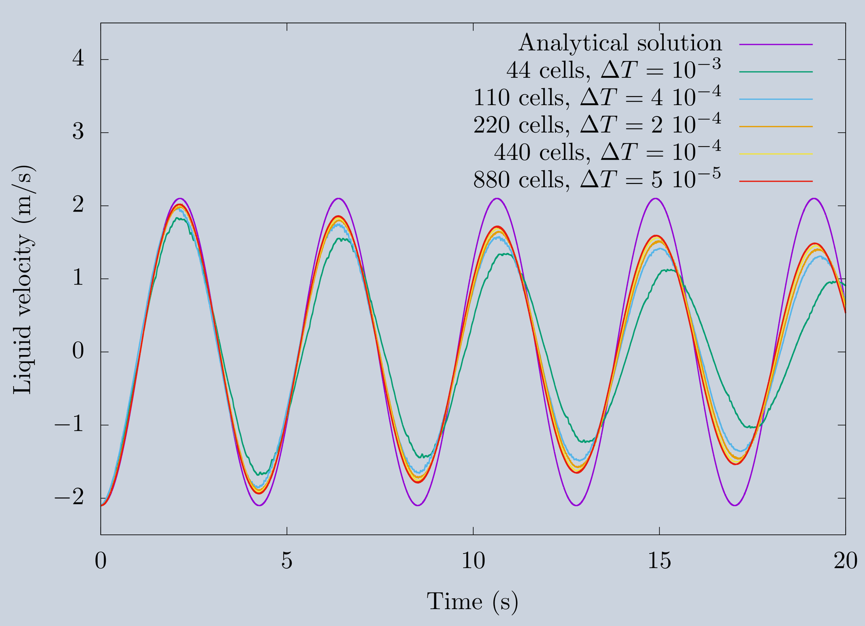


Figure : Numerical convergence for liquid velocity

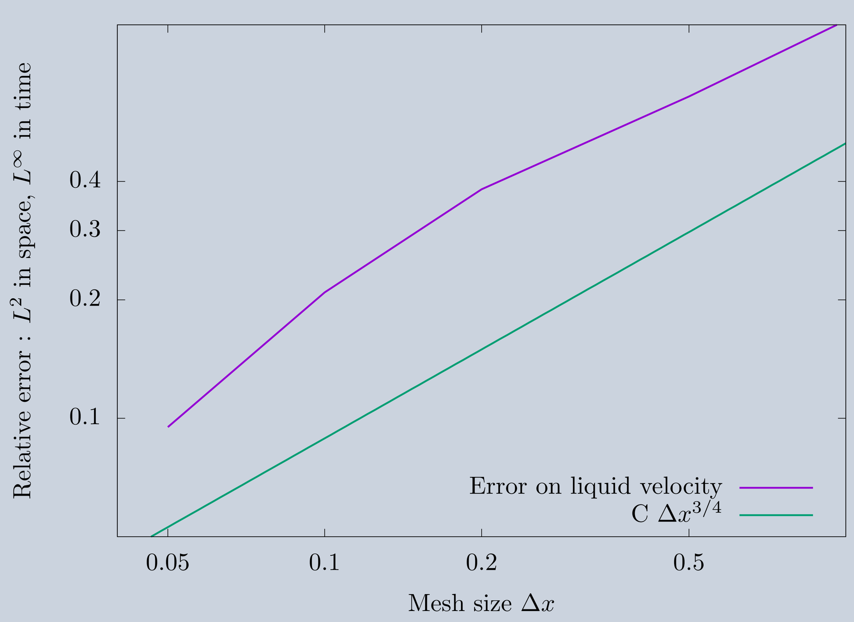


Figure : Fractionnal order of convergence of the Cathare scheme, see [BoucheGhidagliaPascal,2006]

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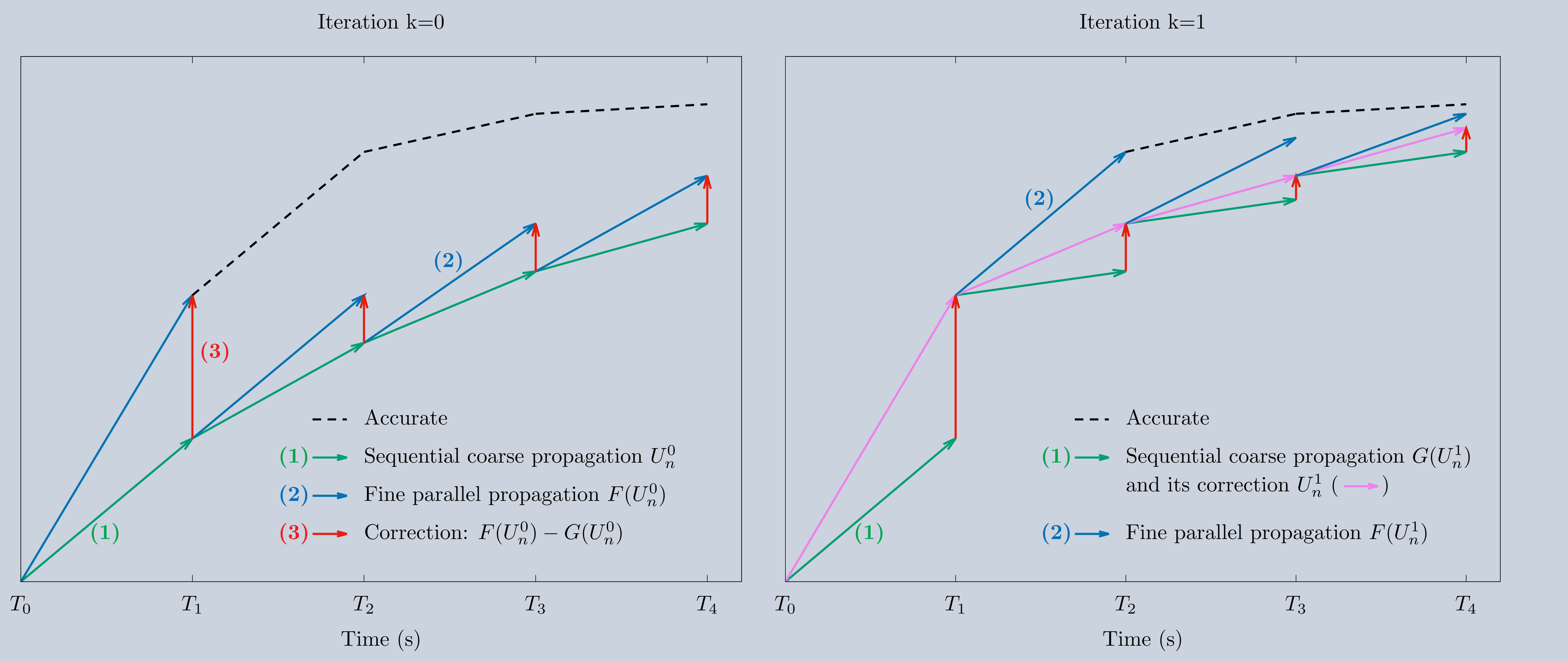
Parareal algorithm



- $\mathbf{U_n^k}$ is obtained by the recurrence relation:

$$U_{n+1}^{k+1} = G_{T_n}^{T_{n+1}}(U_n^{k+1}) + F_{T_n}^{T_{n+1}}(U_n^k) - G_{T_n}^{T_{n+1}}(U_n^k)$$

- Stability and convergence analysis in [GanderVandewalle,2007].
- Instabilities can arise for hyperbolic equations [DaiMaday,2012]

Numerical results

Coarse and fine solvers share the same physics and mesh 110 cells:

- $\mathbf{\Delta t_{coarse} = 2.5 \times 10^{-4}}$ and $\mathbf{\delta t_{fine} = 10^{-5}}$ (left)
- $\mathbf{\Delta t_{coarse} = 10^{-4}}$ and $\mathbf{\delta t_{fine} = 10^{-5}}$ (right)

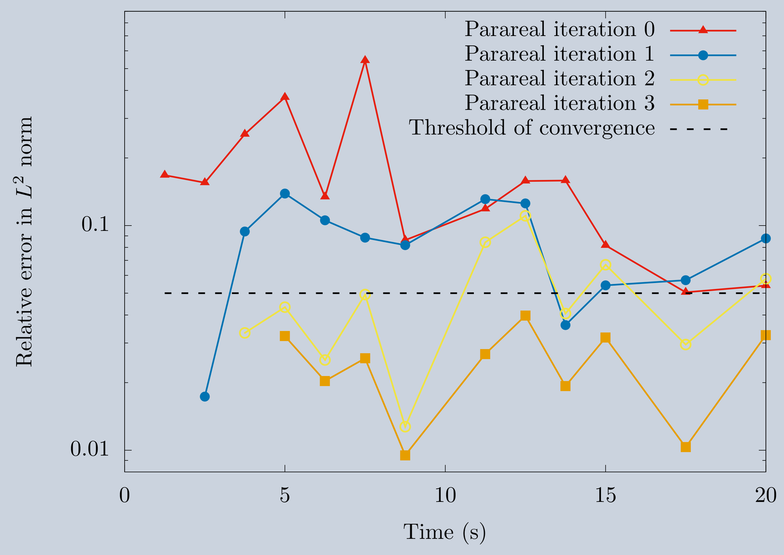


Figure : Convergence of the parareal algorithm: $e^k(T^n) = \frac{||U_n^k - U_n^{seq}||_{L^2}}{||U_n^{seq}||_{L^2}}$, $\forall n \in \{0, 1, \cdots, N\}$, $k \in \{0, 1, 2, 3\}$

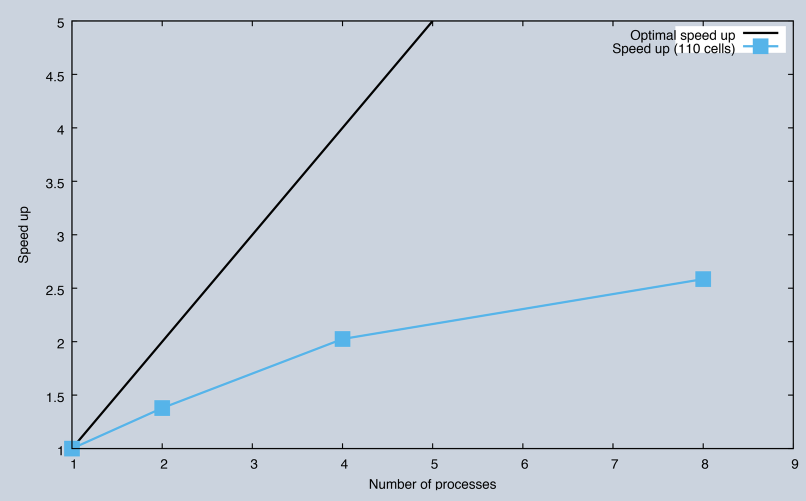


Figure : Speed up (Strong scaling)

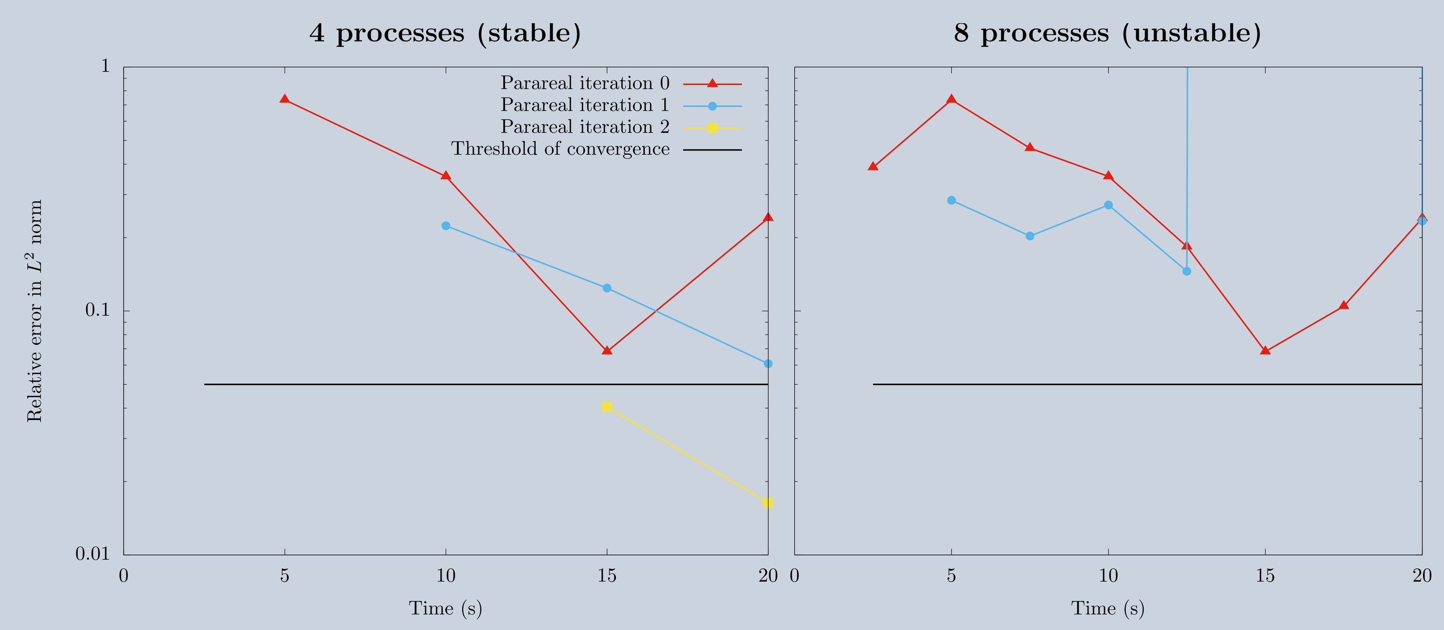


Figure : Parareal with $\mathbf{\Delta T = 4 \times 10^{-4}}$ on 4 and 8 time windows

Conclusions

- Implementation of a numerical clone of Cathare restricted to the oscillating manometer
- Order of convergence of the Cathare scheme
- Numerical convergence and speed up performances
- Influence of coarse solver on the stability of parareal

Perspectives

- Study of the instability of parareal for hyperbolic problems
- Analysis of the properties of Cathare scheme in monophasic case
- Coarsen spatial discretisation also and use projection operators between meshes
- Performances of the method : scalability, scheduling of tasks