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Modelling of bedload sediment transport for weak and strong flow regimes - Nº 2

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Motivation

We focus on bedload transport where particles move by sliding or rolling over the bed. This phenomena is usually described by a Saint-Venant-Exner (SVE) model: a hydrodynamic component for water layer, coupled with a morphodynamic one for the sediment layer.

Main drawbacks of the SVE:

- ✗ sediment discharge is defined empirically
- ✗ the mass conservation is not ensured
- ✗ no energy balance associated
- ✗ no gravitational effects

Goal n°1: Formal derivation of a SVE model from an appropriate asymptotic approximation of NS equations that counteracts these drawbacks.

Goal n°2: To introduce a bilayer Saint-Venant model that generalizes the previous SVE model and converges to it in the bedload regime, i.e., when hydrodynamic time $\ll$ morphodynamic time.

(Goal 1) The proposed SVE model

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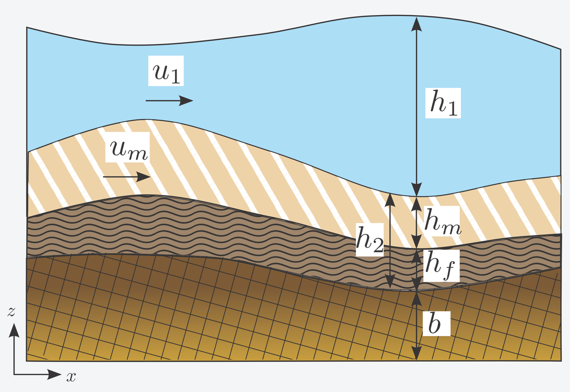
Modelling framework



h_1: thickness of the water layer;
h_2: thickness of the sediment layer;
h_m: thickness of the mobile bed;
h_f: thickness of the fixed bed;
$v_i = (u_i, w_i)$: velocities;
ρ_i: densities; $r = \frac{\rho_1}{\rho_2} < 1$.

Some considerations

- ▷ Governing equations: 3D incompressible Navier-Stokes.
- ▷ "Fluids" with different physical properties.
- ▷ Flows with different behavior (hydrodynamic time $\ll$ morphodynamic time).
- ▷ Interaction through friction laws.
- ▷ Arbitrary sloping beds.

Asymptotic approach

- △ Upper layer: Clear water. $\Rightarrow$ shallow-water equation ($\epsilon = H/L \ll 1$)
- ▽ Lower layer: Bed load. $\Rightarrow$ Reynolds equation (Pressure $\gg$ convection)
- ♯ Different characteristic times $\Rightarrow$ Multiscale analysis in time ($t_{hydro} = \epsilon\, t_{morpho}$)

Resulting model (M_1)

$$\begin{cases} \partial_t h_1 + \mathrm{div}_x q_1 = 0; \\ \partial_t q_1 + \mathrm{div}_x(h_1(u_1 \otimes u_1)) + \frac{1}{2} g \nabla_x h_1^2 + g h_1 \nabla_x (b + h_2) + \frac{g h_m}{r}\mathcal{P} = 0; \\ \partial_t h_2 + \mathrm{div}_x\left(h_m\, v_b\, Q\right) = 0; \\ \partial_t h_f = -T_m \end{cases}$$

with δ : Coulomb angle, θ_c : critical Shields

$$Q = \sqrt{(1/r - 1) g d_s},\quad \vartheta = \frac{\theta_c}{\tan\delta}$$

$$\mathcal{P} = \nabla_x (r h_1 + h_2 + b) + (1 - r) sgn(u_2) \tan\delta$$

$$v_b = \frac{1}{Q} u_1 - \left(\frac{\vartheta}{1-r}\right)^{1/2} |\mathcal{P}|^{1/2} sign(\mathcal{P})$$

- ✓ Discharge obtained explicitly
- ✓ Associated dissipative energy
- ✓ Mass conservation
- ✓ Gravitational effects $\downarrow$

$$\frac{\tau_{\text{eff}}}{\rho_1} = \frac{\vartheta\, d_s\, \tau}{h_m\, \rho_1} - \frac{g d_s \vartheta}{r} \nabla_x (r h_1 + h_2 + b) \quad \text{and} \quad \theta_{\text{eff}} = \frac{|\tau_{\text{eff}}|/\rho_1}{(1/r - 1) g d_s}$$

★ Generalization of MPM model for arbitrary sloping beds

$$\frac{q_b}{Q} = \mathrm{sgn}(\tau) \frac{8}{(1-\varphi)} (\theta - \theta_c)_+^{3/2} \longrightarrow \frac{q_b}{Q} = sgn(\tau_{\text{eff}}) \frac{k_1}{1-\varphi} (\theta_{\text{eff}} - \theta_c)_+^{3/2}$$

(Goal 2) The proposed bilayer bedload model

Submitted to ADWR. https://arxiv.org/abs/1711.03592

Bilayer model (M_2)

$$(M_2)\begin{cases} \partial_t h_1 + \mathrm{div}_x(h_1 u_1) = 0, \\ \partial_t (h_1 u_1) + \mathrm{div}_x(h_1 (u_1 \otimes u_1)) + g h_1 \nabla_x (b + h_1 + h_2) = -F, \\ \partial_t h_2 + \mathrm{div}_x(h_m u_m) = 0, \\ \partial_t (h_m u_m) + \mathrm{div}_x(h_m (u_m \otimes u_m)) + g h_m \nabla_x (b + r h_1 + h_2) \\ \qquad = r F + \frac{1}{2} u_m T_m - (1 - r) g h_m sgn(u_m) \tan\delta, \end{cases}$$

★ Key point: **Friction term**

$$F = C_Q (u_1 - u_m)|u_1 - u_m|; \quad C_Q = \frac{1}{\alpha}\frac{h_1 h_m}{\vartheta (h_1 + h_m)}; \quad \alpha = \begin{cases} k_{\max} d_s & h_m \le k_{\max} d_s \\ h_m & h_m > k_{\max} d_s \end{cases}$$

Advantages

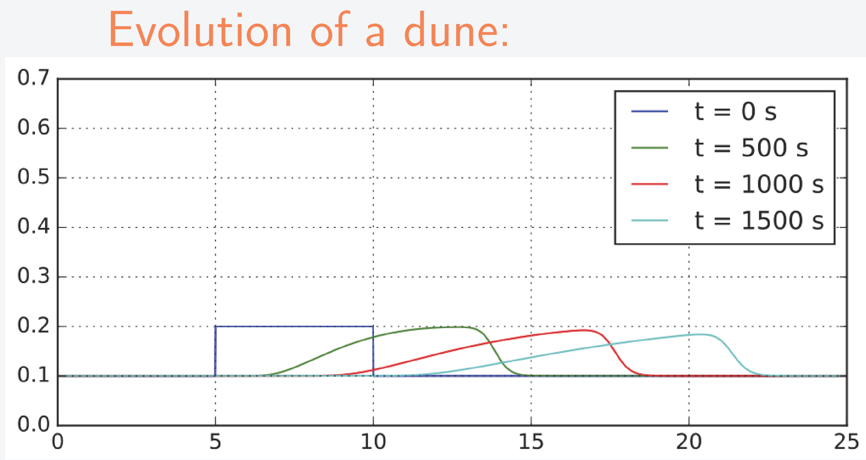
- ★ Avoid high numerical cost in approximation of gravitational effects
- ★ "Automatic" bedload regime behavior
- ★ $(M_2) \longrightarrow (M_1)$ when $t_{hydro} \ll t_{morpho}$ ($\Leftrightarrow u_{morpho} \ll u_{hydro}$)
- ★ Non-hydrostatic effects may be incorporated.

Numerical results

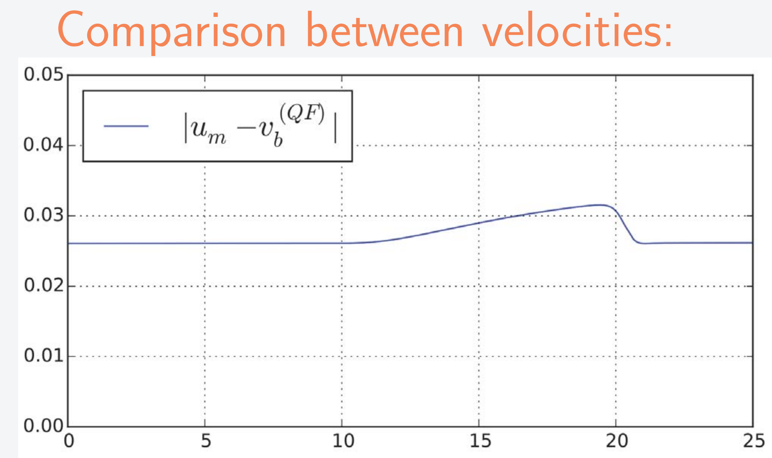
Test 1: Weak bedload transport (Dune)

Initial height: $h_1 + h_2 = 1m$; left boundary condition $q_1(t, 0) = 1 m^2/s^2$.

Evolution of a dune:



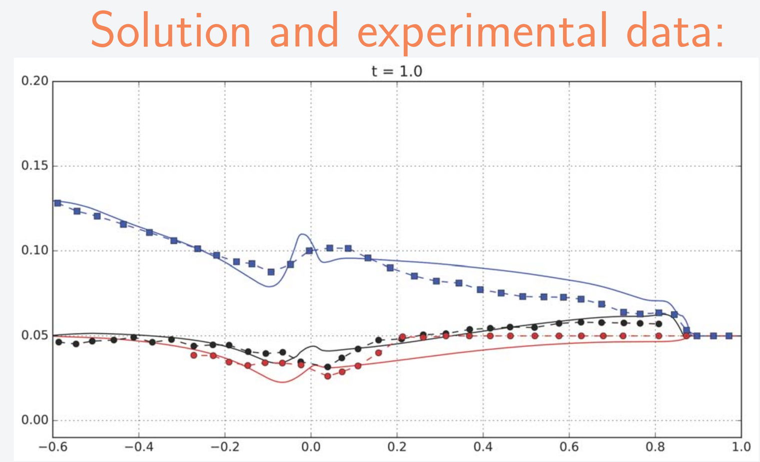
Comparison between velocities:



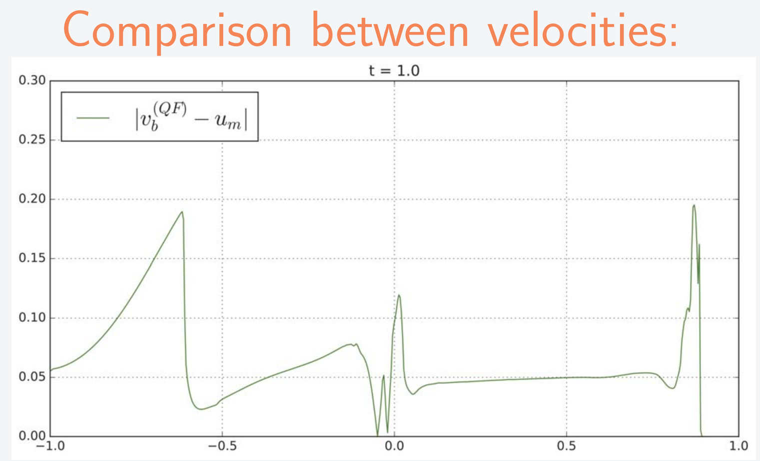
Test 2: Strong bedload transport (Dam-break problem)

Initial condition: $h_1 = 0.10m$ if $x < 0$ and $h_2 = 0.05m$

Solution and experimental data:



Comparison between velocities:



Conclusions

- ▶ The bilayer model gives promising results for weak and strong bedload transport regimes.
- ▶ It converges to the SVE model thanks to the definition of the friction terms.
- ▶ It is a good choice from a computational point of view for bedload problems.

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