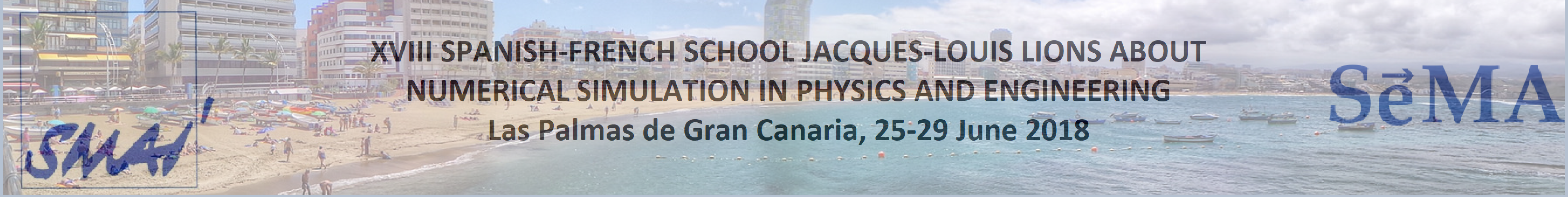

UNAMalla: a system for generating quality structured meshes on irregular plane regions - Nº 24

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Abstract

UNAMalla is a system for generating quality structured meshes on irregular plane regions. The grids generated by UNAMalla have different geometric properties which can be combined according the problem where they will be used. The meshes are generated via a non-linear large-scale optimization procedure, starting from an initial mesh which is often non convex, until a smooth and convex grid is obtained.
In this new version has added tools to: a) distribute points on the boundaries b) improvement the distribution of the areas of the cells c) geometric adaptation d) quality measures e) B-spline parametrization for IGA.

Introduction

Definition of a discrete structured mesh. Let B be the unit square and $U(m,n)$ the uniform mesh of size $m \times n$ on B given by

$$U(m,n) = \left\{ \left(\frac{i}{m}, \frac{j}{n} \right) \mid \quad 0 \le i \le m, 0 \le j \le n \right\}$$

where

$$\partial U(m,n) = \partial B \cap U(m,n)$$

A discrete grid G of size $m \times n$ on Ω is a mapping

$$G : U(m,n) \mapsto \mathbf{R}^2$$

such that

$$G(\partial U) \subset \partial \Omega$$

and

$$\partial G = G(\partial U) = \Gamma_{mn} \subset \partial \Omega$$

In the follow figure 1 we can see this idea.

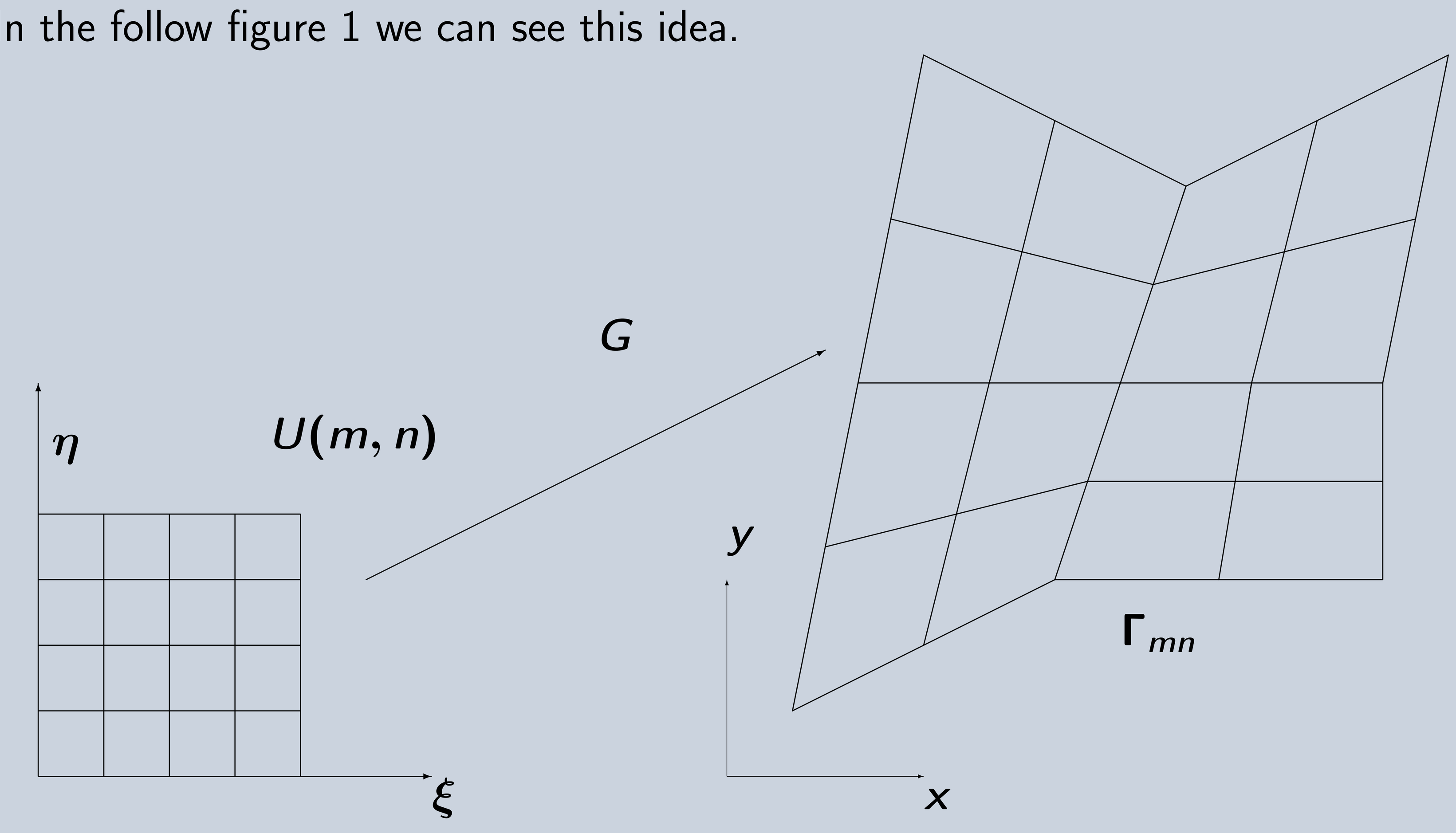


Figure: A discrete structured mesh.

If the bondary of Ω is positively oriented, an orientation of the boundary of the grid cell $c_{ij} = G(B_{ij})$ is induced in a natural way and in an analogous way the orientation of the boundary of the four triangles defined by the grid vertices

Area's convex functional $S_{\omega,\epsilon}$

An efficient solution to the problem was proposed by Barrera and Domínguez-Mota [5], who posed the next theorem, which provides a characterization of the functionals whose minima are convex grids.

Theorem Let bet $0 < \epsilon \le 1$. If f is a C^2 strictly decreasing convex and bounded below function such that $f(\alpha) \to 0$ as $\alpha \to \infty$, then

$$S_{\omega,\epsilon}(G) = \sum_{q=1}^{N} f(\omega\alpha(\triangle_q) - \epsilon\overline{\alpha}(G)) \qquad (1)$$

can be used as the objective function of the optimization problem, whose optimal grids are ϵ-convex is $\omega > 0$ is large enough.

The functionals $S_{\omega,\epsilon}$ have an infinite *barrier* at the boundary of the grid set ϵ-convex $\mathcal{D}_\epsilon \subset \mathcal{A}_\epsilon$. This means that at least one triangle's area is very close to ϵ when G is kept inside $\mathcal{D}_\epsilon$; then for ω large enough $S_{\omega,\epsilon} \longrightarrow \infty$, see [5].

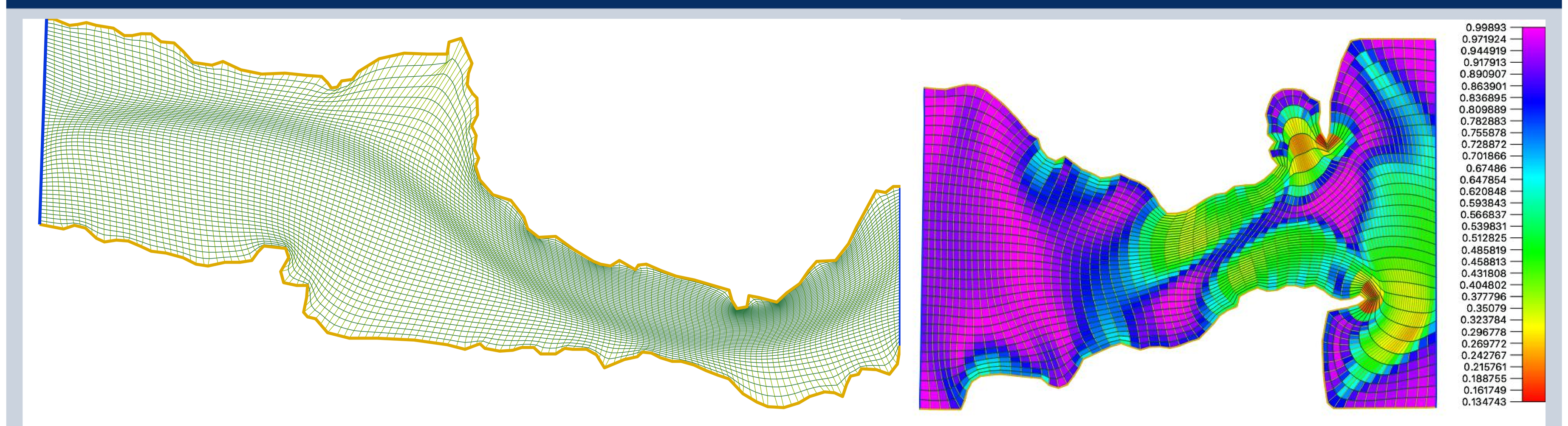
Grids with preset cell size distribution

Give an ϵ-convex grid G_0, there exists $\omega > 0$ such that the grid G^* satisfies

$$G^* = \arg\min_{G \in \mathcal{A}_\epsilon} \sum_{q=1}^{N} [(1-\sigma) f(\omega\alpha(\triangle_q) - \epsilon\overline{\alpha}(G)) + \sigma w_q f_c(\triangle_q)] \qquad (2)$$

The last theorem, indicates that using positive weights it is posible to generate an ϵ-convex grid for ω large enough, see [3].

Geometric adaptation and quality grids



B-spline parametrization for IGA

We assume that $\mathbf{x}$ is a *biquadratic B-spline map* that can be written as

$$\mathbf{x}(\xi,\eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} \mathbf{P}_{i,j} B^3_{i,t^\xi}(\xi) B^3_{j,t^\eta}(\eta) \qquad (3)$$

where $\mathbf{P}_{i,j} = (P^x_{i,j}, P^y_{i,j})^t$, $i = 1,\dots,n$, $j = 1,\dots,m$ are the control points. The function $B^3_{i,t^\xi}(\xi)$ is the i-th quadratic B-spline for the knot sequence t^ξ and the function $B^3_{j,t^\eta}(\eta)$ is the j-th quadratic B-spline for the knot sequence t^η with

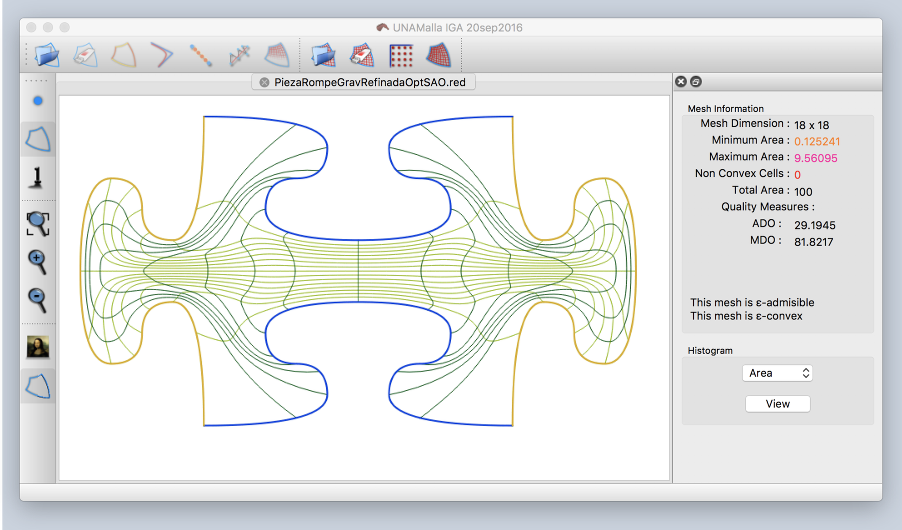
$$t^\xi = (0,0,\xi_1,\xi_2,\dots,\xi_{n-1},1,1) \qquad (4)$$

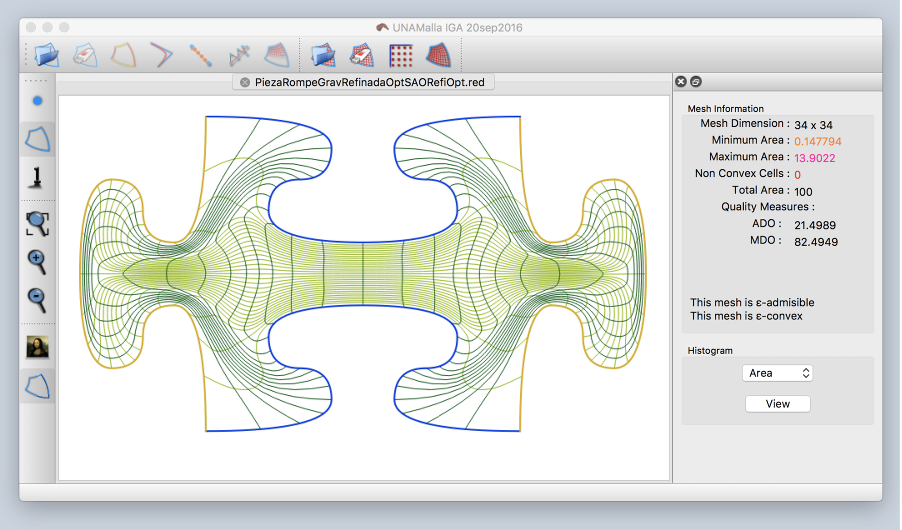
$$t^\eta = (0,0,\eta_1,\eta_2,\dots,\eta_{m-1},1,1) \qquad (5)$$

and $\xi_i \equiv \frac{i-1}{n-2}$, $i = 1 \dots n-1$ and $\eta_j \equiv \frac{j-1}{m-2}$, $j = 1 \dots m-1$.

Then, the interior control points $\mathbf{P}_{i,j}$, $i = 2,\dots,n-1$, $j = 2,\dots,m-1$ of $\mathbf{x}(\xi,\eta)$ are computed as the inner vertices of a *structured quadrilateral mesh* G, we compute those points minimizing a functional $F(G)$ that measures some geometric properties of G, , see [1].

B-spline parametrization for IGA





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