XVIII SPANISH-FRENCH SCHOOL JACQUES-LOUIS LIONS ABOUT
NUMERICAL SIMULATION IN PHYSICS AND ENGINEERING
Las Palmas de Gran Canaria, 25-29 June 2018

4

Reduced Basis Method for natural convection in a variable height cavity

T. Chacón Rebollo, E. Delgado Ávila, M. Gómez Mármol

Dpto. Ecuaciones Diferenciales y Análisis Numérico

Reduced Basis Model

We define the steady Boussinesq VMS-Smagorinsky model as follows:

$$\begin{cases} (\mathbf{u}\cdot\nabla\mathbf{u},\mathbf{v})_{\Omega_o} + Pr(\nabla\mathbf{u},\nabla\mathbf{v})_{\Omega_o} - (p,\nabla\cdot\mathbf{v})_{\Omega_o} - Pr\,Ra(\theta,v_d)_{\Omega_o} & \\ +(\nu_T(\Pi_h^*\mathbf{u})\nabla(\Pi_h^*\mathbf{u}),\nabla(\Pi_h^*\mathbf{v}))_{\Omega_o} = \langle\mathbf{f},\mathbf{v}\rangle & \forall\mathbf{v}\in Y(\Omega_o(\mu_g)) \\ (\nabla\cdot\mathbf{u},q)_{\Omega_o} = 0 & \forall q\in M(\Omega_o(\mu_g)) \\ (\mathbf{u}\cdot\nabla\theta,z)_{\Omega_o} + (\nabla\theta,z)_{\Omega_o} + \frac{1}{Pr}(\nu_T(\Pi_h^*\mathbf{u})\nabla(\Pi_h^*\theta),\nabla(\Pi_h^*z))_{\Omega_o} = \langle Q,z\rangle & \forall z\in\Theta(\Omega_o(\mu_g)), \end{cases} \quad (1)$$

where $\nu_T(\Pi_h^*\mathbf{u}) = (C_S h_K)^2|\nabla(\Pi_h^*\mathbf{u})|\chi_K$, and $\Pi_h^* = Id - \Pi_h$, with Π_h an interpolation operator.
In order to have parameter-independent matrices in the online phase, we define a transformation map $T:\Omega_r\times\mathscr{D}\to\mathbb{R}^d$ such that $\Omega_o = T(\Omega_r;\mu_g)$. The map considered in this case is:

$$T((x,y);\mu_g) = \begin{pmatrix}1 & 0\\ 0 & \mu_g\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}, \quad \forall(x,y)\in\Omega_r.$$

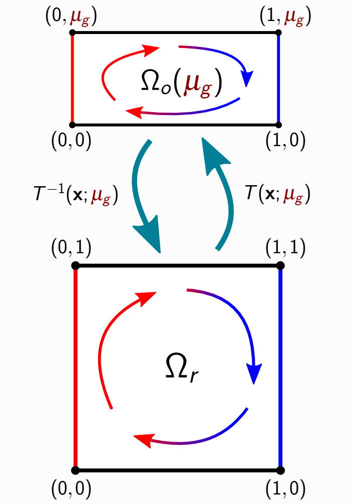
We transform Problem (1) into an equivalent one, taking into account the change of variable formula. For example, the diffusion term is transformed in the following way

$$Pr(\nabla\mathbf{u},\nabla\mathbf{v})_{\Omega_o} = Pr\mu_g\left[(\partial_x u_1,\partial_x v_1)_{\Omega_r} + (\partial_x u_2,\partial_x v_2)_{\Omega_r}\right] + \frac{Pr}{\mu_g}\left[(\partial_y u_1,\partial_y v_1)_{\Omega_r} + (\partial_y u_2,\partial_y v_2)_{\Omega_r}\right].$$

The reduced basis spaces are given by

$$Y_N = \text{span}\{\zeta_{2k-1}^{\mathbf{v}} := \mathbf{u}(\mu_g^k),\ \zeta_{2k}^{\mathbf{v}} := T_p^{\mu_g}\xi_k^p,\ \ k=1,\dots,N\},$$
$$M_N = \text{span}\{\xi_k^p := p^u(\mu_g^k),\ \ k=1,\dots,N\},\quad \Theta_N = \text{span}\{\varphi_k^\theta := \theta(\mu_g^k),\ \ k=1,\dots,N\}.$$

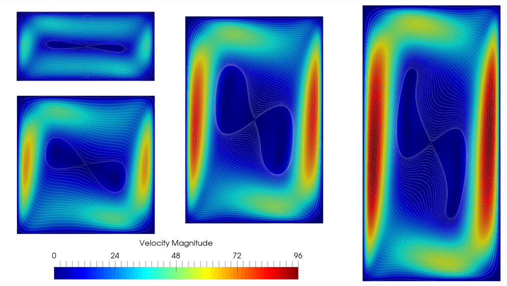
Transformation map



Numerical Results: $\mu_g \in [0.5, 2]$

We solve the RB Boussinesq VMS-Smagorinsky model in a cavity, for $\mu_g\in[0.5,2]$. We consider a regular mesh with $h = 0.02\sqrt{2}$. The EIM selects $M = 72$ basis functions for the approximation of the eddy viscosity, and the Greedy algorithm selects $N = 32$ basis functions.

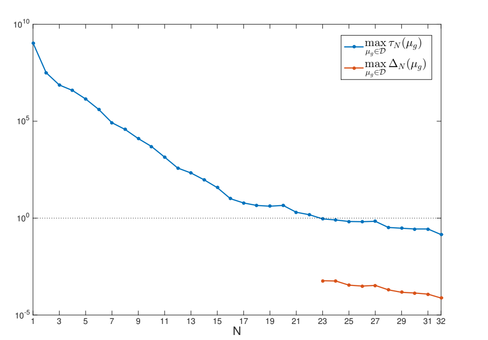
- Snapshots for different values of μ_g: $\mu_g = 0.5$, $\mu_g = 1$, $\mu_g = 1.5$, $\mu_g = 2$.

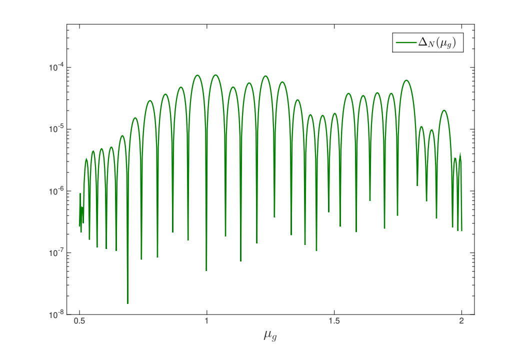


Data	$\mu_g = 0.64$	$\mu_g = 1.08$	$\mu_g = 1.44$	$\mu_g = 1.87$
T_{FE}	808.91s	810.16s	866.1s	851.82s
T_{online}	2.68s	2.55s	2.61s	2.52s
speedup	301	317	331	337
$\|\mathbf{u}_h-\mathbf{u}_N\|_1$	$1.13\cdot10^{-6}$	$1.86\cdot10^{-6}$	$2.82\cdot10^{-6}$	$3.4\cdot10^{-6}$
$\|\theta_h-\theta_N\|_1$	$6.28\cdot10^{-8}$	$8.83\cdot10^{-9}$	$9.37\cdot10^{-9}$	$9.48\cdot10^{-9}$
$\|p_h-p_N\|_1$	$1.69\cdot10^{-5}$	$3.7\cdot10^{-5}$	$8.35\cdot10^{-5}$	$8.82\cdot10^{-5}$

Table: Computational time for FE and RB solutions, with the speedup and the error.

- (Left) Maximum of the *a posteriori* error bound estimator in the Greedy algorithm.
- (Right) *A posteriori* error bound estimator value at $N = N_{\max} = 32$.





Bibliography & Acknowledgments

- T. Chacon, E. Delgado, M. Gomez, F. Ballarin, G. Rozza, *On a certified Smagorinsky reduced basis turbulence model.* SIAM J. Numer. Anal. Vol. 55, No. 6, pp. 3047-3067 (2017).
- S. Deparis, G. Rozza, *Reduced basis method for multi-parameter-dependent steady Navier-Stokes equations: Applications to a natural convection in a cavity.* J. Comp. Physics 339: 4359-4378 (2009).
- F. Hecht, *New development in FreeFem++.* J. Numer. Math. **20**, no. 3-4, 251–265 (2012).

This work has been supported by Spanish Government Project MTM2015-64577-C2-1-R and COST Action TD1307.