XVIII SPANISH-FRENCH SCHOOL JACQUES-LOUIS LIONS ABOUT NUMERICAL SIMULATION IN PHYSICS AND ENGINEERING
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Tumor development model with vasculature

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Introduction

Nowadays one of the worst brain disease is the brain cancer. In this work, we present a model about glioblastoma, which is the most frequent malignant primary brain tumor and it has a very high mortality, with a median survival of 14.6 months fot those patients receiving standard care. This model includes as variable the vasculature, that influence on the speed of the tumor diffusion. We also show some medical outcomes that we check with our model with numerical simulations in different situations.

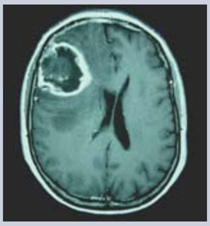

Figure: Glioblastoma

Model

$$\begin{cases} \dfrac{\partial T}{\partial t} = \nabla \cdot (D(\phi)\nabla T) + \rho(\phi)\ T\left(1 - \dfrac{T+N+\Phi}{K}\right) - \alpha\, f(\phi)\ T - \beta\, N\, T \\ \dfrac{\partial N}{\partial t} = \alpha\, f(\phi)\ T + \beta\, N\, T + \delta\, T\, \Phi + \beta\, N\Phi \\ \dfrac{\partial \Phi}{\partial t} = \gamma\, f(\phi)\dfrac{T}{K}\Phi\left(1 - \dfrac{T+N+\Phi}{K}\right) - \delta\, T\, \Phi - \beta\, N\, \Phi \end{cases}$$

$T \rightarrow$ Tumor
$N \rightarrow$ Necrosis
$\Phi \rightarrow$ Vasculature
$\phi = \dfrac{\Phi}{T+\Phi}$,
$f(\phi) = \sqrt{1-\phi^2}$,
$D(\phi) = \bar{D}\,\phi, \quad \bar{D} \in \mathbb{R}^+$
$\rho(\phi) = \bar{\rho}\,\phi, \quad \bar{\rho} \in \mathbb{R}^+$

Variable	Description	Value
$D(\phi)$	Diffusion Speed	cm^2/day
$\rho(\phi)$	Proliferation rate	day^{-1}
β	Change rate to necrosis influence	day^{-1}
α	Hypoxic death rate by persistent anoxia	day^{-1}
γ	Vasculature proliferation rate	day^{-1}
δ	Vasculature death by tumor action	day^{-1}
K	Carriying capacity	$cell/cm^3$

Simulations with Vasculature piecewise constant

$$T_0 = \chi_{\tilde{T}}, \qquad \tilde{T} = \left\{(x,y) \in \mathbb{R}^2 \colon \frac{x^2}{2} + y^2 \leq 0.05,\ x^2+y^2 \geq 0.01\right\}$$

$$N_0 = \chi_{\tilde{N}}, \qquad \tilde{N} = \{(x,y) \in \mathbb{R}^2 \colon x^2+y^2 < 0.01\}$$

$$\Phi_0 = \left((0.03)\,\chi_{[0,3]^2} \cup (0.06)\,\chi_{[-3,0]\times[0,3]} \cup (0.09)\,\chi_{[-3,0]^2} \cup (0.12)\,\chi_{[0,3]\times[-3,0]}\right)$$

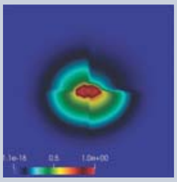

Figure: Necrosis

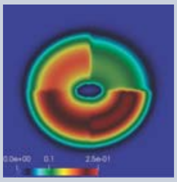

Figure: Tumor

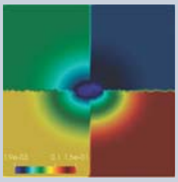

Figure: Vasculature

Simulations with Vasculature regular

$$T_0 = \chi_{\tilde{T}}, \qquad \tilde{T} = \{(x,y) \in \mathbb{R}^2 \colon x^2+y^2 \leq 0.8,\ x^2+y^2 \geq 0.01\}$$

$$N_0 = \chi_{\tilde{N}}, \qquad \tilde{N} = \{(x,y) \in \mathbb{R}^2 \colon x^2+y^2 < 0.01\}$$

$$\Phi_0 = \left\{(x,y) \in [0,3]\times[0,3] \colon y = \frac{x^2}{100}\right\}$$

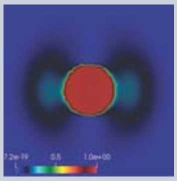

Figure: Necrosis

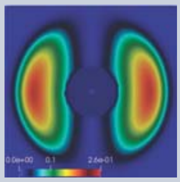

Figure: Tumor

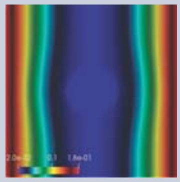

Figure: Vasculature

Tumor Rim width-Volume relation with the parameter α

In this picture we see the survival time of patients with two types of glioblastomas:

- One of them with a narrow tumor rim (line grease).
- Another with a width tumor rim (line black).

and we check that this behaviour is connected with the parameter α of our model.

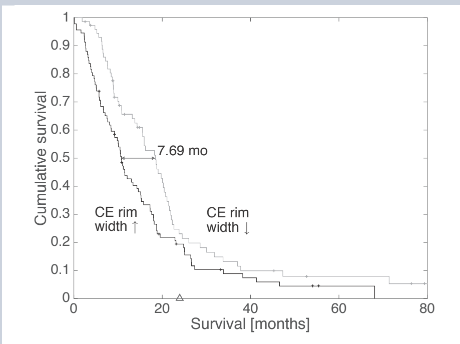

Figure: Survival respect tumor rim width

Figure: $\alpha = 500$

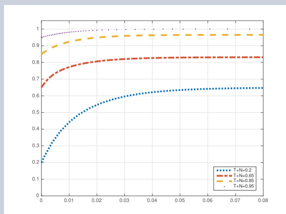

Figure: T+N for $\alpha = 500$

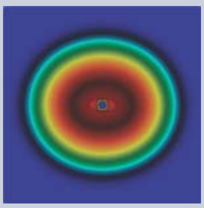

Figure: $\alpha = 1$

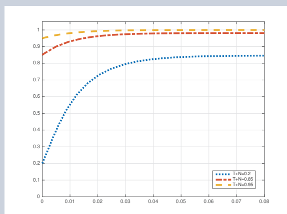

Figure: T+N for $\alpha = 1$

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