Multiobjective Evolutionary Algorithms for Engineering Optimum Design

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Evolutionary Algorithms

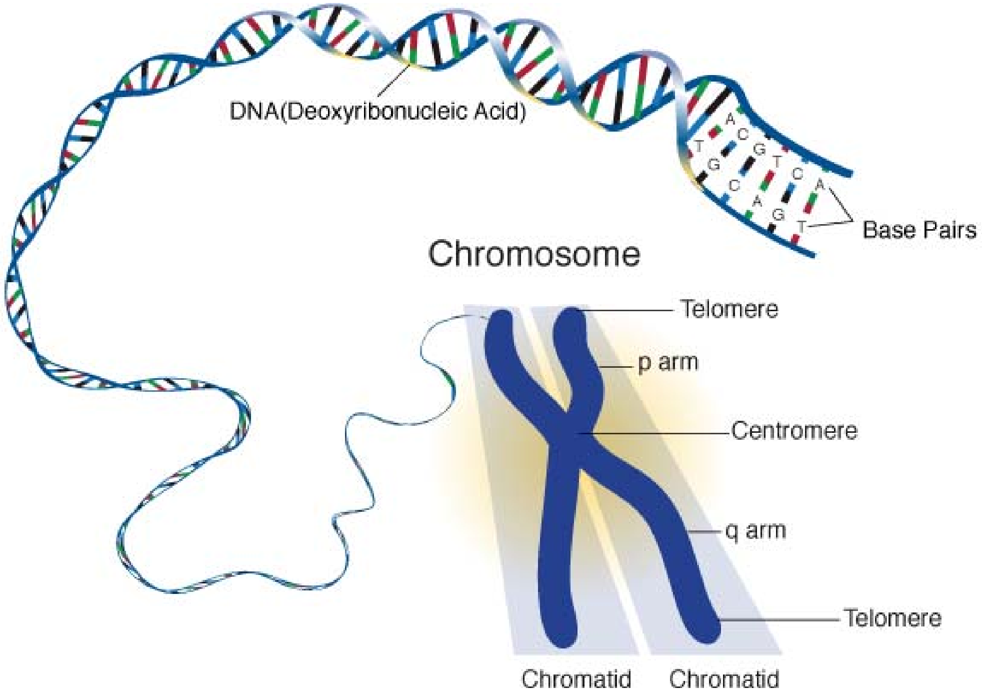
Evolutionary Algorithms (alternatively also called *Evolutionary Computation*) are based on the use of Darwinian notions of inheritance and natural selection in a computationally useful form.

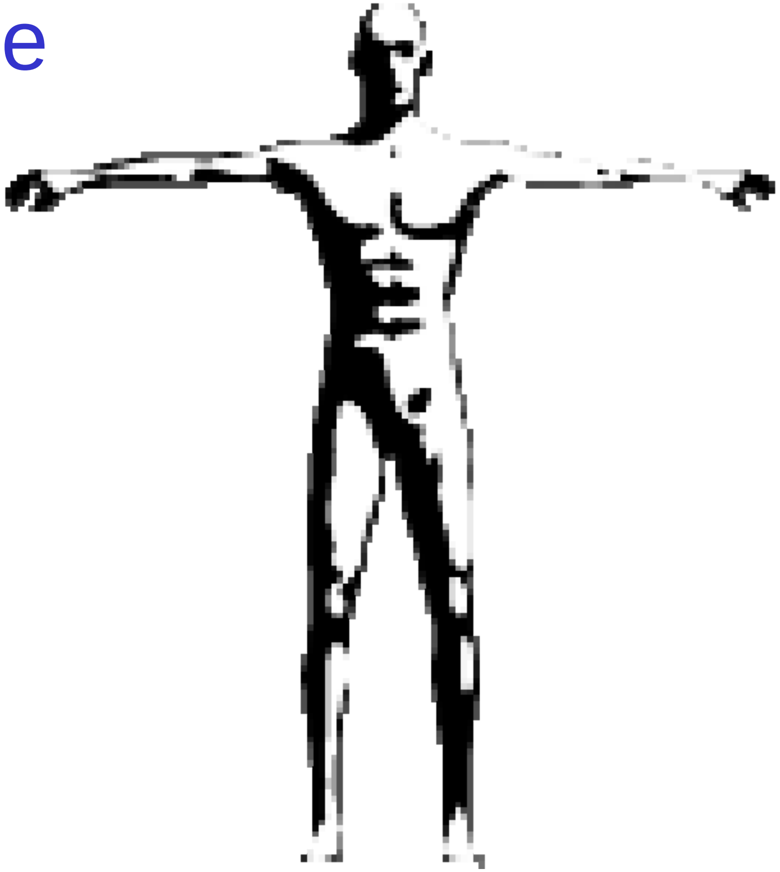
They use evolutionary processes to solve difficult computational problems creating good solution candidates in an automatic way.

They are search and optimisation tools based on stochastics approaches.

Evolutionary Algorithms: Natural Evolution

Genotype versus Phenotype





Evolutionary Algorithms

The success of EAs has been proved in a variety of applications, including problems that can hardly be solved through traditional optimisation methods.

EAs may equally handle single and multi-objective which are likely to involve more than one discipline.

EAs only require evaluation of the function in search space points, the convergence is not affected by the continuity or differentiability of the function to be optimised in the applications

Evolutionary Algorithms & Metaheuristics

Among EAs & Metaheuristics, the following algorithms are included:

- Genetic Algorithms (GAs)
- Evolution Strategies (ES)
- Differential Evolution (DE)
- Genetic Programming (GP)
- Particle Swarm Optimization (PSO)
- Others (Estimation of Distribution Algorithms EDAs, Ant Colony Optimization ACO, etc.) …

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Multiobjective Optimization

In engineering optimising a problem with more than one criteria is a frequent necessity.

There are functions in conflict, where the improvement in one criteria implies the worsening in another objective.

There is not only a single optimal solution, but a set of optimal solutions called *Pareto frontier.*

With this set of solutions, is the designer or engineer mission, to choose the most suitable solution according to her or his requirements and preferences.

Pareto Domination / Pareto Frontier

The Italian economist Vilfredo Pareto (1848-1923),

postulated the efficient mode of resource allocation, which bears his name:

‘Resources are allocated efficiently in the Pareto sense when unable to improve the welfare of any person without worsening the other'.

Pareto Domination / Pareto Frontier

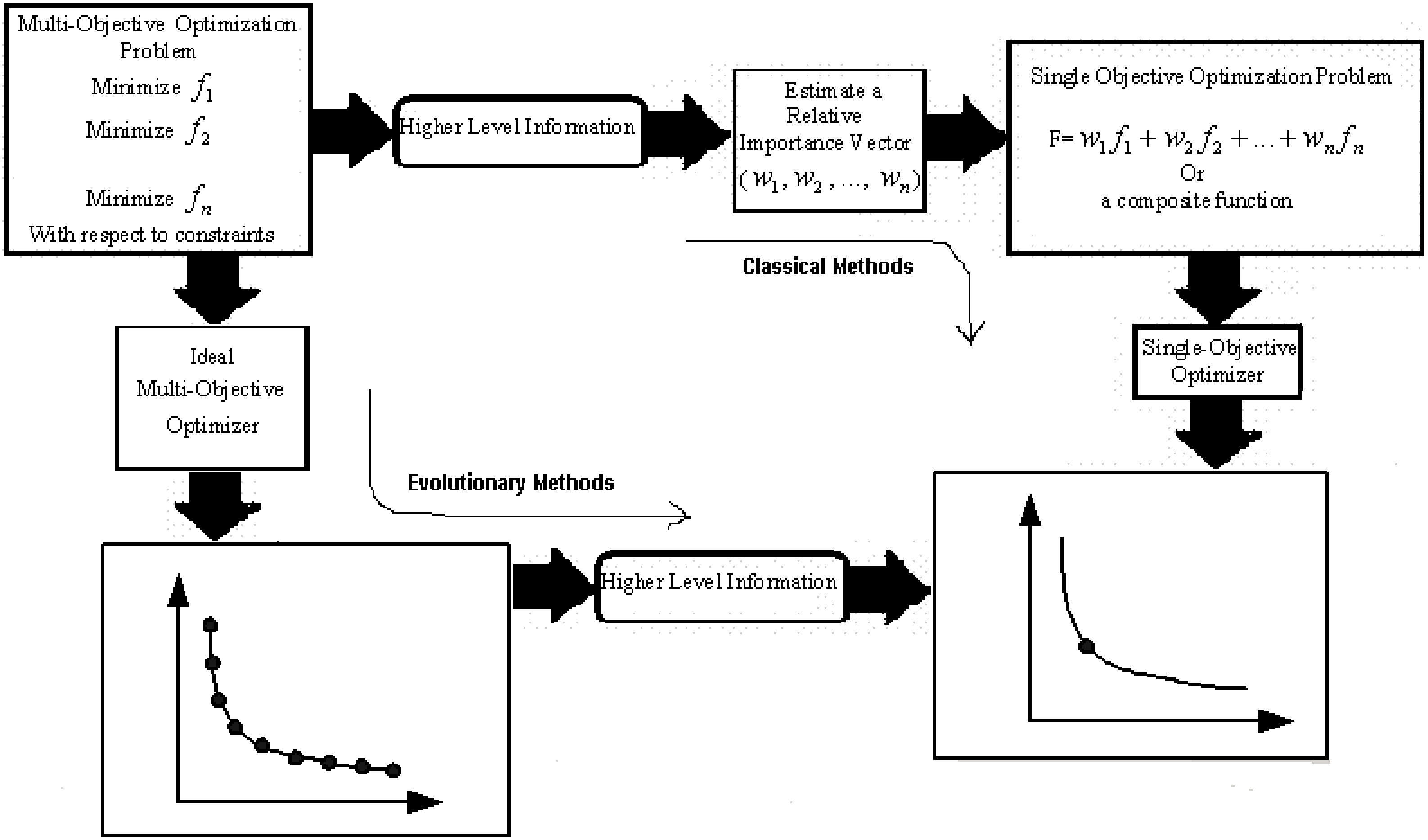
A x solution is non-dominated if:

1) The *x* solution is worse than *y* in all the criteria. For two minimised criteria, it means that *x* is less or equal than *y* in both.

2) The *x* solution is strictly better than *y* at least in one criterium. For two minimised criteria, it means that *x* is less than *y* in at least one objective.

In the set of all possible solutions, the non-dominated ones, constitute the Pareto Frontier.

Multiobjective Optimization



Multi-Objective Optimization Problem

Minimize f_1

Minimize f_2

Minimize f_n

With respect to constraints

Higher Level Information

Estimate a Relative Importance Vector $(w_1, w_2, \ldots, w_n)$

Single Objective Optimization Problem

$$F = w_1 f_1 + w_2 f_2 + \ldots + w_n f_n$$

Or

a composite function

Classical Methods

Single-Objective Optimizer

Ideal Multi-Objective Optimizer

Evolutionary Methods

Higher Level Information

From: K. Deb “Multi-objective optimization using Evolutionary Algorithms”, 2001, Wiley.

Classical Methods Disadvantages

Among the disadvantages of the traditional multiobjective methods respect to the multiobjective evolutionary algorithms we have the followings [Deb, EUROGEN99]:

Many times should be applied a traditional multiobjective algorithm in order to obtain multiple non-dominated solutions, because of only one solution is found each application.

They can require some information about the handled problem -for example, the method of pondering coefficients in order to determine the values of the parameters-,

They can be sensitive to the Pareto front shape, not capable to find solutions located in certain zones like non-convex ones.

The spread of the Pareto solutions found depends on the efficiency of the mono-criteria optimizer used.

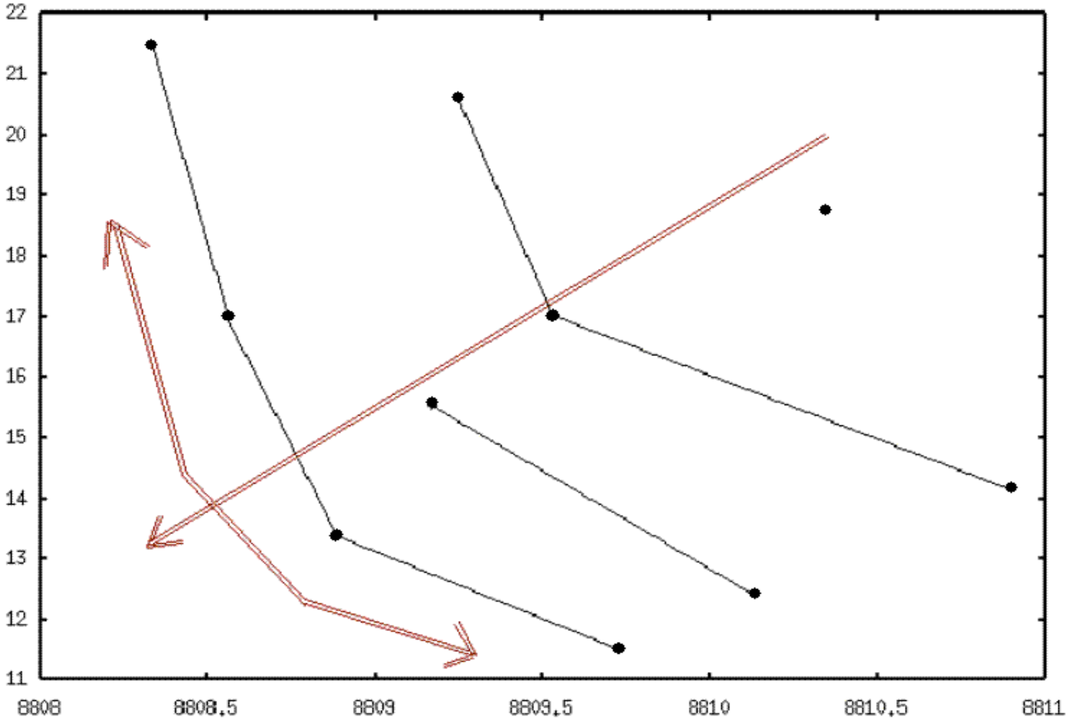
They are not appropriate in problems with stochasticities or uncertainties.

They can not handle problems with discrete domain.

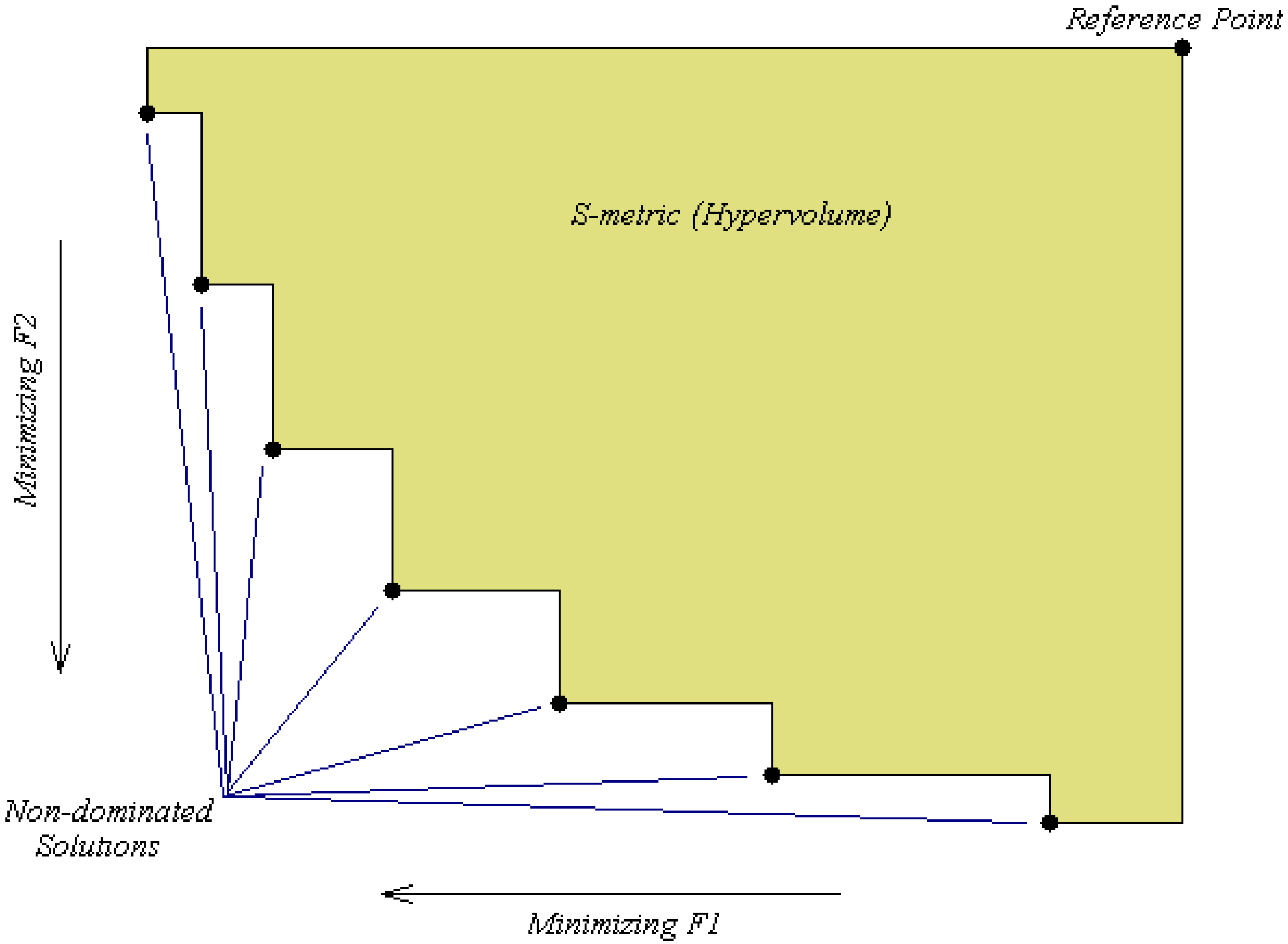
Evolutionary Multiobjective Algorithms

A multicriteria algorithm should be capable to satisfy two requeriments in order to obtain appropriate results:

1. To conduct the search towards the Pareto Frontier;
2. To maintain the diversity of the population along this front.



Hypervolume / S-metric



Evolutionary Multiobjective Algorithms

We can classify EMO in three groups:

Dominance based selection EMO: Use the concept of Pareto dominance as the basis of their selection (e.g., NSGA2, SPEA2), based on the suggestion of David Goldberg in 1989 proposing the use of the Pareto dominance criterion to perform multiobjective optimization.

Indicator based selection EMO: Based on some unary indicator to guide the search. The main indicator used is the hypervolume indicator, as in e.g., SMS-EMOEA, HypE.

Decomposition/Aggregated based selection EMO Methods based on decomposition of the search space, optimizing a set of scalarizing functions in parallel (MOEA/D, Global WASF-GA).

Test Cases

Three Test Cases have been selected and solved with NSGA2 (all of them are biobjective, both minimizing functions), each representing one of the difficulties that classical optimization methods have, and which evolutionary multiobjective optimization ones can surpass:

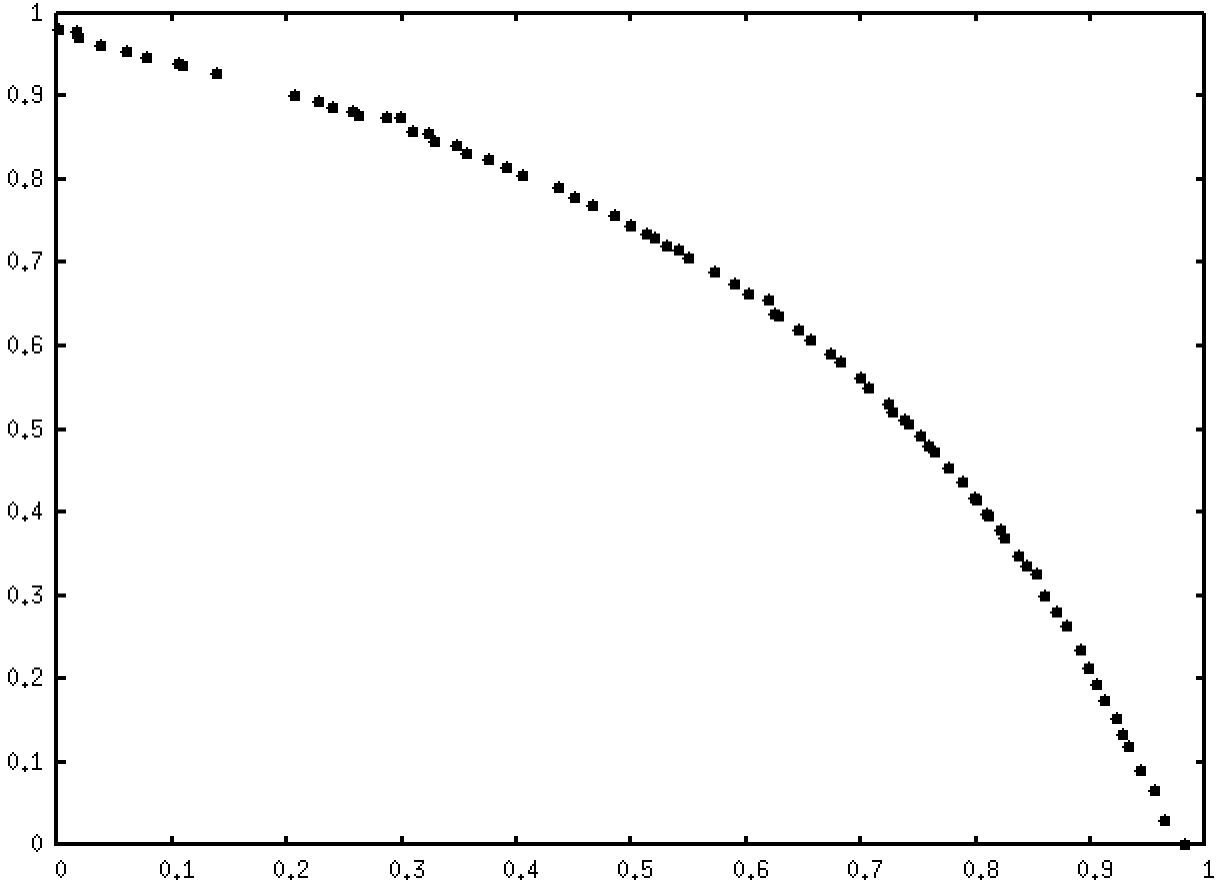
1. **Continous Non-Convex** Pareto Front: from D. Van Veldhuizen and Gary B. Lamont. "Multiobjective Evolutionary Algorithm Test Suites", *Proceedings of the 1999 ACM Symposium on Applied Computing*, pages 351-357, San Antonio, Texas,. ACM (1999).

2. **Discontinous** Pareto Front: from C.A. Coello Coello, "Multiobjective Optimization using a Micro-Genetic Algorithm", *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2001)*, San Francisco, Morgan Kaufmann.

3. **Global and Local** Pareto Front: from K. Deb, "Multi-Objective Genetic Algorithms: Problem Difficulties and Constructions of Test Problems", *Evolutionary Computation* 7-3 (1999) pp. 205-230. MIT Press.

Test Cases

Continuous Non-Convex Pareto Front: from D. Van Veldhuizen and Gary B. Lamont. "Multiobjective Evolutionary Algorithm Test Suites", *Proceedings of the 1999 ACM Symposium on Applied Computing*, pages 351-357, San Antonio, Texas,. ACM . (1999).



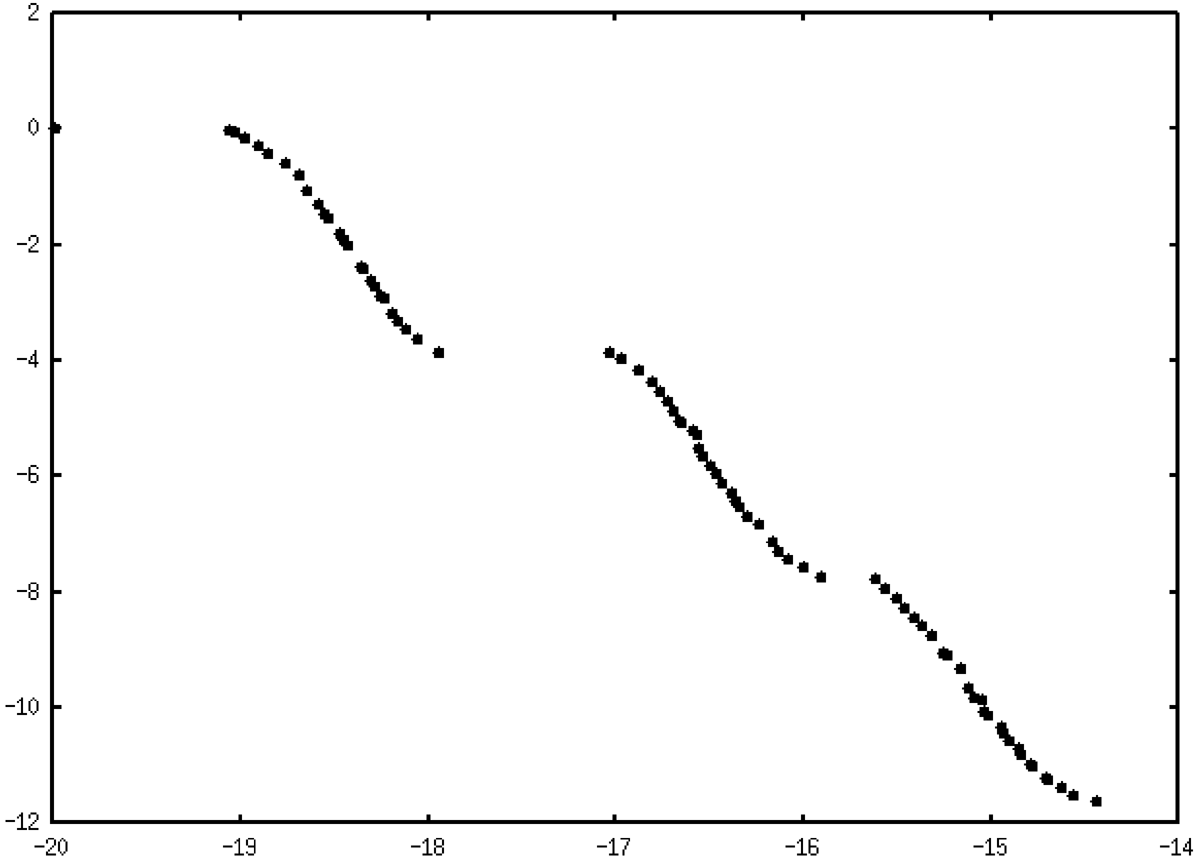
$$\min f_1(\mathbf{x}) = 1 - e^{-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2}$$

$$\min f_2(\mathbf{x}) = 1 - e^{-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2}$$

$$-2 \le x_1, x_2 \le 2$$

Test Cases

Discontinous Pareto Front: from C.A. Coello Coello, “Multiobjective Optimization using a Micro-Genetic Algorithm”, *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2001)*, San Francisco, Morgan Kaufmann.



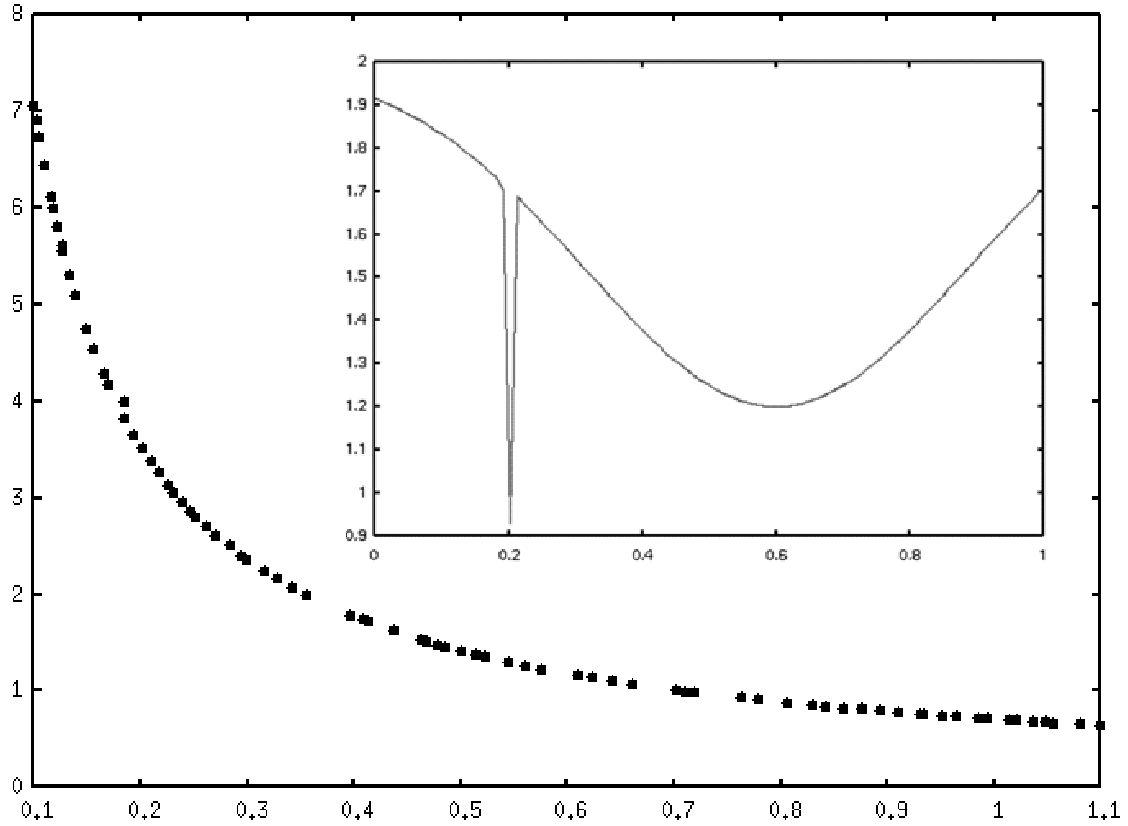
$$min\ f_1(\mathbf{x}) = \sum_{i=1}^{n-1}\left(-10\cdot e^{\left(-2\sqrt{x_i^2+x_{i+1}^2}\right)}\right)$$

$$min\ f_2(\mathbf{x}) = \sum_{i=1}^{n}|x_i|^{0.8} + 5\cdot sin(x_i^3)$$

$$-5 \le x_1, x_2, x_3 \le 5$$

Test Cases

Global and Local Pareto Front: from K. Deb, “Multi-Objective Genetic Algorithms: Problem Difficulties and Constructions of Test Problems”, *Evolutionary Computation* 7-3 (1999) pp. 205-230. MIT Press.



$$min\ f_1(\mathbf{x}) = x_1$$

$$min\ f_2(\mathbf{x}) = \frac{g(x_2)}{x_1}$$

$$g(x_2) = 2 - e^{\left[-\left(\frac{x_2-0.2}{0.004}\right)^2\right]} - 0.8 \cdot e^{\left[-\left(\frac{x_2-0.6}{0.4}\right)^2\right]}$$

$$0.1 \leq x_1 \leq 1.1\ ;\ 0 \leq x_2 \leq 1$$

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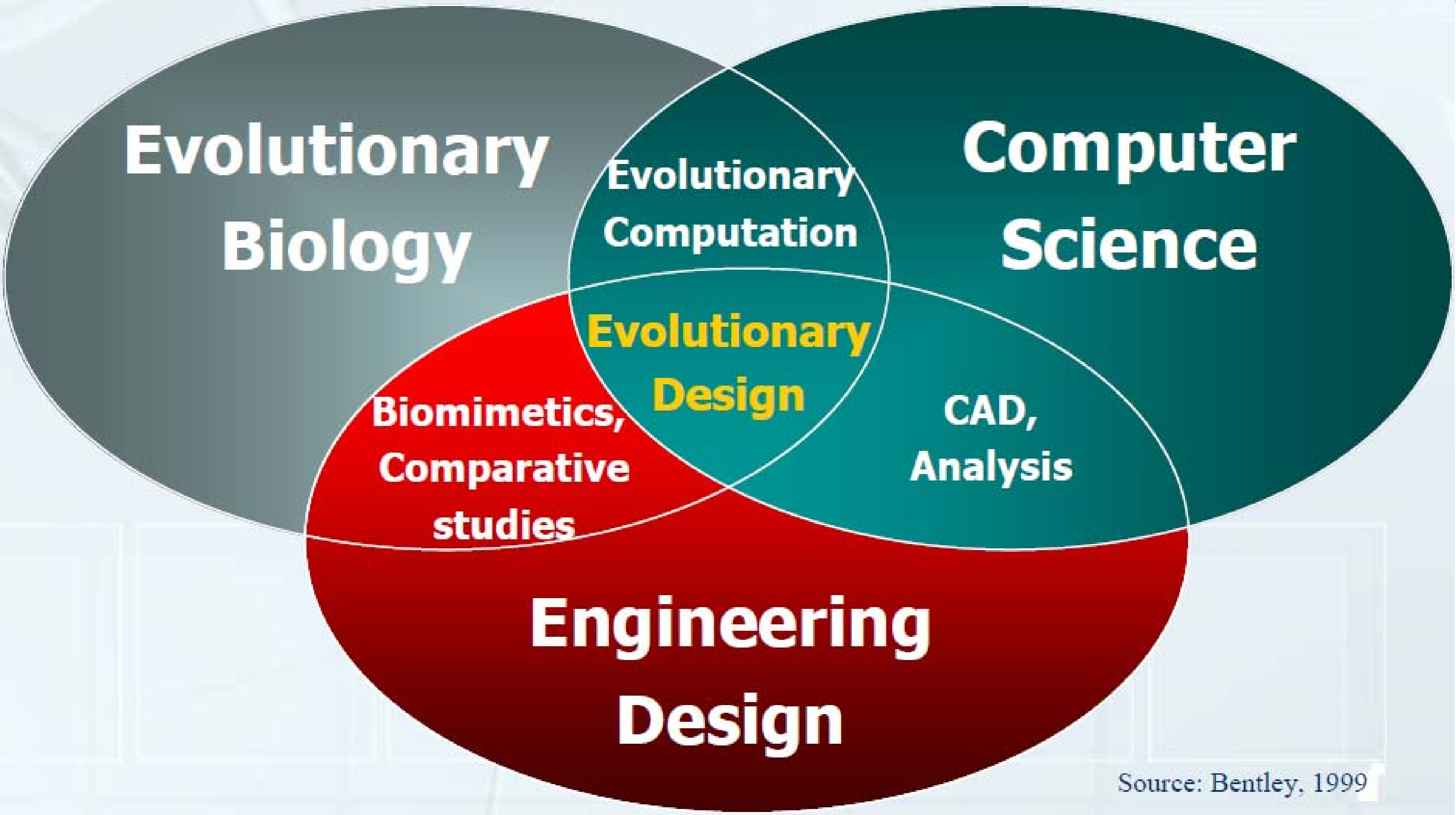
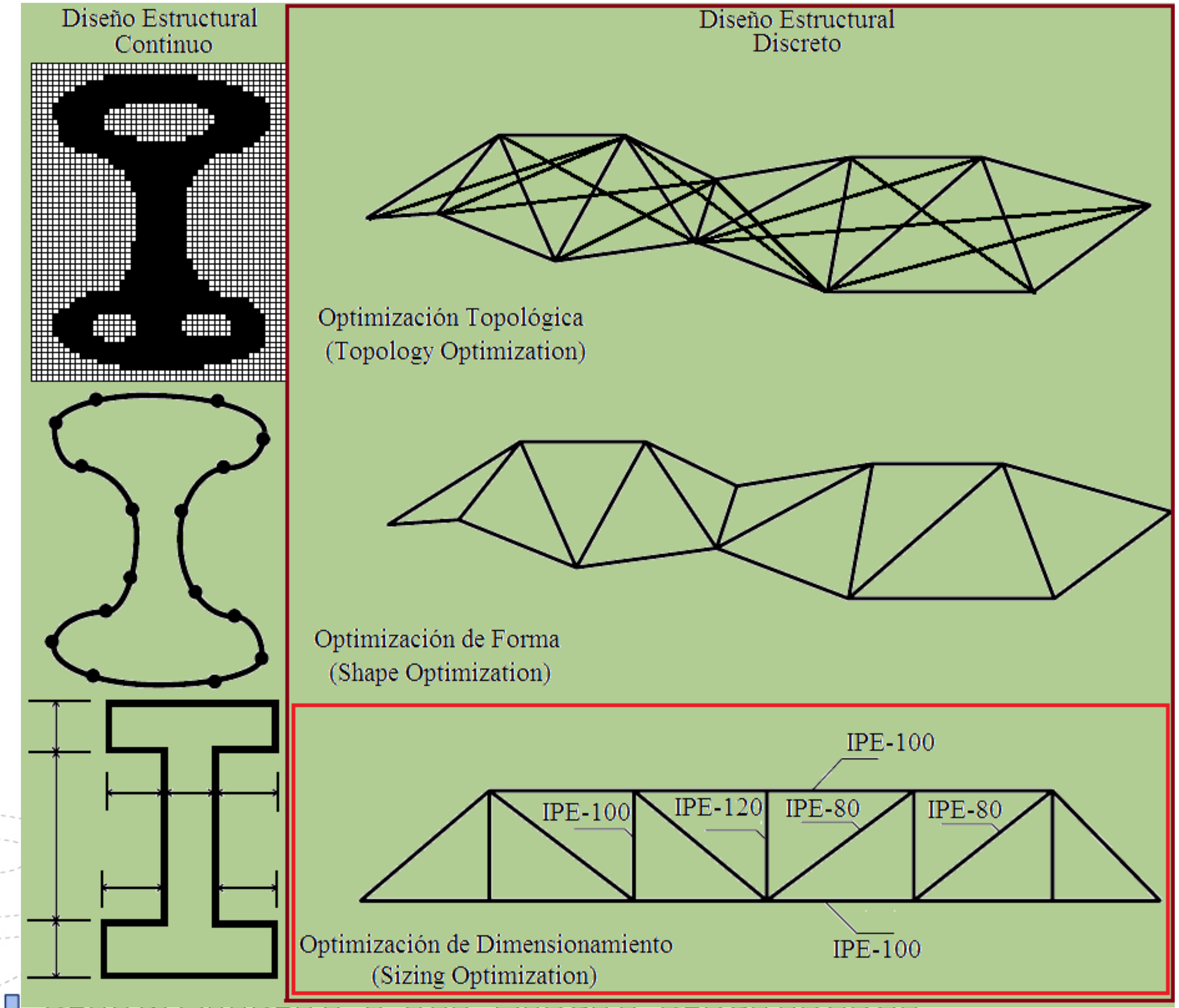
Evolutionary Design



Source: Bentley, 1999

From: R. Kicinger, 2006 – Evolutionary Design

Introduction: Problem handled



Sizing Optimization for
Discrete Skeletal Structures Design

Structural Problem

Using the C/C++ language, the following computational implementation are developed:

- Evaluator: *Frame matrix calculator* *Program* (direct stiffness method), for Bar Structures.
- Optimizer: *Evolutionary Algorithms* (various strategies of multiobjective optimization algorithms).
- *Objective Functions*: Definition (constrained weight and number of different cross-section types).

Chromosome

Genotype versus Phenotype

11111110000000000001111000000000

Objective Function I

1. The *constrained weight*, due to minimize the acquisition cost of raw material of the metallic frame; the following constraints are applied:

- *Stresses* of the bars (usual value for steel structures is the yield limit stress, of 2600 kgp/cm2), for each bar

$$\sigma_{co} - \sigma_{lim} \leq 0$$

- *Compressive slenderness limit*, (buckling effect) compression lambda lower than 200 (limit is dependendent on national codes), for each bar

$$\lambda - \lambda_{lim} \leq 0$$

- *Displacements of joints or middle points* of bars (at each degree of freedom) in certain points, nodes of the beams

$$u_{co} - u_{lim} \leq 0$$

Objective Function I

Resulting the *fitness function constrained weight* the following:

$$Fitness = \left[\sum_{i=1}^{Nbars} A_i \cdot \rho_i \cdot l_i \right] [1 + k \cdot \sum_{j=1}^{Nviols} (viol_j - 1)]$$

where:

A_i = area of cross-section i

p_i = density of bar i

l_i = length of bar i

k = constant that regulates the cocient between constraint and weight.

$viol_j$ = for each of the violated constraints, is the cocient between the violated value (stress, displacement or slenderness) and its reference limit.

Objective Function II

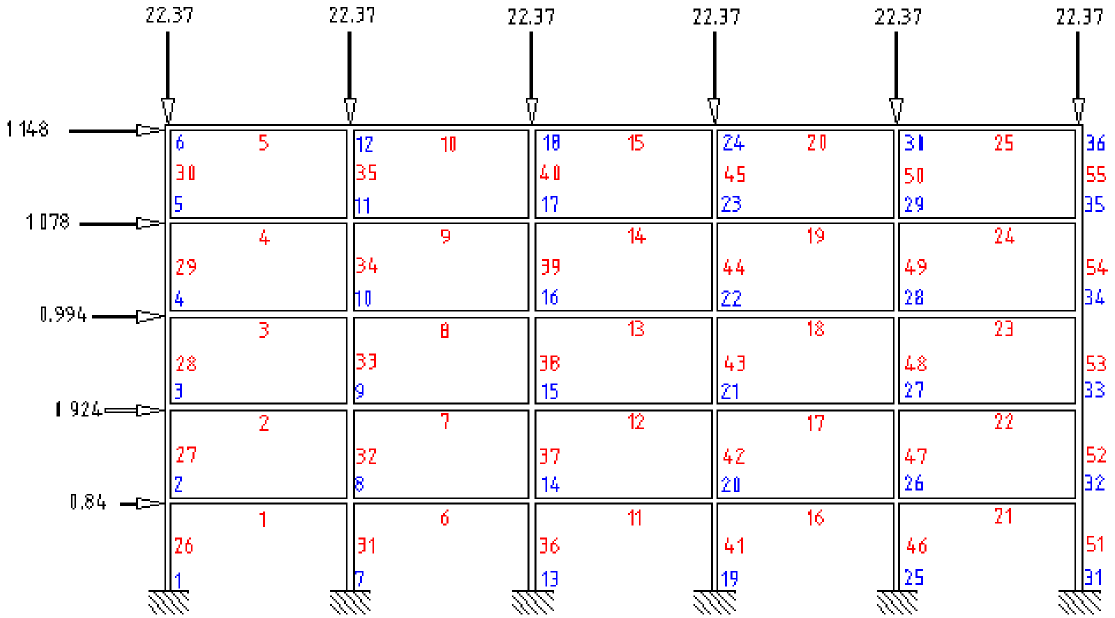
The second fitness function is the *minimization of number of different cross section types*, and its calculation of the quantity is done by successive comparisons of the existing cross-section types in a certain structure.

This factor has been related recently with the structure life cost cycle in Sarma and Adeli (2002), and also in Liu et al (2003).

It corresponds to a constructive requirement, helping a better quality control during the execution of the building site.

Test Case Y

Computational domain, boundary conditions, loadings and design variable set groupings:



Fixed Supports

Based on (**D. Greiner, Emperador, Winter;** ***Computer Methods in Applied Mechanics and Engineering, Elsevier, 2004***)

Figure includes elements and nodes numbering, and punctual loads in tons.

Every beam supports a uniform load of 39,945 N/m.

Maximum vertical displacement in each beam is l/300 = 1.867 cm.

Test Case Y

- IPE cross section types for beams (set between IPE-080 and IPE-500)
- HEB for columns (set between HEB100 and HEB-450)
- Admissible stresses of 2.2 and 2.0 T/cm^2 for beams and columns, respectively.
- Density and elasticity modulus E (steel) are: 7.85 T/m^3 and 2100 T/cm^2.
- Based on a continuous variable reference test problem of S. Hernández.
- The span is 5.6 m and the height of columns is 2.80 m.

55 members

Search Space: $16^{55} = 2^{4x55} = 2^{220} = 1{,}7{\cdot}10^{66}$

Experimental Cases

Thirty independent Executions per case

Three population sizes: 50, 100 and 200 individuals

Uniform crossover; Rate= 100%

Uniform Mutation;

Four mutation rates: 0.4%, 0.8%, 1.5%, 3%

Two Codifications: Binary and Gray

Case Y (200.000 evaluations per case)

Experimental Cases

Whole number of evaluated structures Test Case Y:

30 x 3 x 4 x 2 x 200.000 = 144 million ≈ $14.4 \cdot 10^{7}$

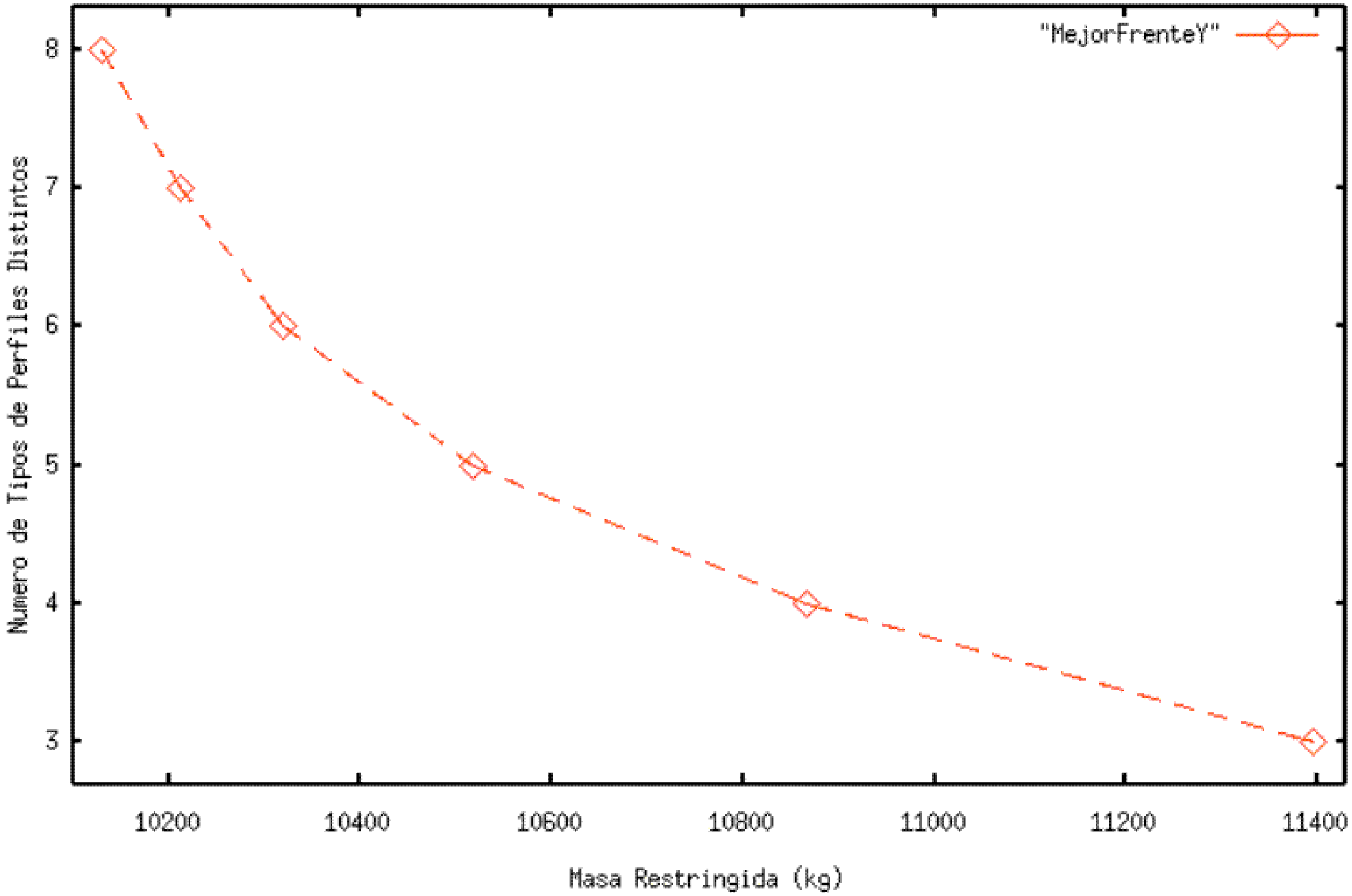
Search Space Dimension $1,7 \cdot 10^{66}$ structures

Observable Universe Mass: $3 \cdot 10^{55}$ g.

(more stars in the universe than grains of sand in Earth beaches:
100.000 millons of stars in each of the 100.000 millons of galaxies)

Test case Y : we explore 2.5 mg over whole universe mass !!!

Test Case II: Pareto Front



D. Greiner, Emperador, Winter; ***Computer Methods in Applied Mechanics and Engineering, Elsevier, 2004***

Results Test Case Y (Discrete Optimization)

Número de tipos de perfiles distintos (F1)	8
Masa (kg)	10127.29
Barra nº 1	IPE330
Barra nº 2	IPE330
Barra nº 3	IPE330
Barra nº 4	IPE300
Barra nº 5	IPE330
Barra nº 6	IPE300
Barra nº 7	IPE300
Barra nº 8	IPE300
Barra nº 9	IPE300
Barra nº 10	IPE300
Barra nº 11	IPE300
Barra nº 12	IPE300
Barra nº 13	IPE300
Barra nº 14	IPE300
Barra nº 15	IPE300
Barra nº 16	IPE300
Barra nº 17	IPE300
Barra nº 18	IPE300
Barra nº 19	IPE300
Barra nº 20	IPE300
Barra nº 21	IPE300
Barra nº 22	IPE300
Barra nº 23	IPE300
Barra nº 24	IPE300
Barra nº 25	IPE300

Tensión Barra nº 1	1939.3
Tensión Barra nº 2	1869.2
Tensión Barra nº 3	1876.3
Tensión Barra nº 4	2188.7
Tensión Barra nº 5	1800.3
Tensión Barra nº 6	2065.1
Tensión Barra nº 7	2047.2
Tensión Barra nº 8	2009.8
Tensión Barra nº 9	2026.7
Tensión Barra nº 10	2165.4
Tensión Barra nº 11	2129.1
Tensión Barra nº 12	2073.8
Tensión Barra nº 13	2047.6
Tensión Barra nº 14	2036.9
Tensión Barra nº 15	2049.3
Tensión Barra nº 16	2166.0
Tensión Barra nº 17	2053.8
Tensión Barra nº 18	2057.9
Tensión Barra nº 19	2021.2
Tensión Barra nº 20	2071.5
Tensión Barra nº 21	2024.9
Tensión Barra nº 22	1940.1
Tensión Barra nº 23	1999.4
Tensión Barra nº 24	2000.0
Tensión Barra nº 25	2118.1

Barra nº 26	HEB160
Barra nº 27	HEB180
Barra nº 28	HEB160
Barra nº 29	HEB140
Barra nº 30	HEB180
Barra nº 31	HEB220
Barra nº 32	HEB200
Barra nº 33	HEB180
Barra nº 34	HEB160
Barra nº 35	HEB120
Barra nº 36	HEB200
Barra nº 37	HEB200
Barra nº 38	HEB160
Barra nº 39	HEB140
Barra nº 40	HEB120
Barra nº 41	HEB220
Barra nº 42	HEB200
Barra nº 43	HEB160
Barra nº 44	HEB140
Barra nº 45	HEB120
Barra nº 46	HEB220
Barra nº 47	HEB200
Barra nº 48	HEB160
Barra nº 49	HEB140
Barra nº 50	HEB120
Barra nº 51	HEB180
Barra nº 52	HEB200
Barra nº 53	HEB200
Barra nº 54	HEB160
Barra nº 55	HEB200

Tensión Barra nº 26	1906.9
Tensión Barra nº 27	1849.4
Tensión Barra nº 28	1994.3
Tensión Barra nº 29	1972.9
Tensión Barra nº 30	1854.0
Tensión Barra nº 31	1848.3
Tensión Barra nº 32	1830.0
Tensión Barra nº 33	1745.2
Tensión Barra nº 34	1610.0
Tensión Barra nº 35	1779.1
Tensión Barra nº 36	2000.5
Tensión Barra nº 37	1681.5
Tensión Barra nº 38	1959.8
Tensión Barra nº 39	1900.4
Tensión Barra nº 40	1626.0
Tensión Barra nº 41	1754.1
Tensión Barra nº 42	1710.2
Tensión Barra nº 43	1990.0
Tensión Barra nº 44	1944.3
Tensión Barra nº 45	1650.7
Tensión Barra nº 46	1770.7
Tensión Barra nº 47	1716.1
Tensión Barra nº 48	1991.7
Tensión Barra nº 49	1953.0
Tensión Barra nº 50	1692.8
Tensión Barra nº 51	1983.1
Tensión Barra nº 52	1882.4
Tensión Barra nº 53	1754.8
Tensión Barra nº 54	1888.9
Tensión Barra nº 55	1791.9

D. Greiner, Emperador, Winter; *Computer Methods in Applied Mechanics and Engineering, Elsevier, 2004*

Results Test Case Y (Discrete Optimization)

Real model:

GA Optimum Solution

Weight = 9551.75 kgp.

Constraint =0.13 kgp

Bar Num	Area 1 GA Ideal Model P=9327 kgp	Area 2 GA Real Model P=9551 kgp	Area 2 Real Model Stress (kgp/cm^2)	Area 2 Ideal Model Stress (kgp/cm^2)	Central Deflection Área 2 (cm)
1	56.84	57.00	2196.95	2175.35	0.7417
2	55.04	55.30	2198.70	2177.78	0.7381
3	54.94	55.07	2199.16	2177.94	0.8482
4	52.41	52.79	2198.40	2177.07	0.8583
5	53.28	53.78	2197.67	2175.43	0.9640
6	51.64	51.99	2196.55	2176.74	0.6069
7	51.21	51.60	2199.58	2179.79	0.6310
8	50.47	51.00	2198.30	2178.31	0.6241
9	50.50	50.48	2199.53	2178.76	0.6518
10	52.68	52.82	2199.82	2177.34	0.5513
11	52.15	52.41	2198.07	2177.87	0.6218
12	51.48	51.81	2199.88	2179.90	0.6365
13	51.23	51.53	2198.83	2178.40	0.6515
14	50.89	51.06	2199.61	2178.74	0.6615
15	51.17	51.26	2198.53	2177.39	0.6737
16	52.92	53.15	2200.02	2179.89	0.5817
17	51.29	51.81	2199.70	2179.92	0.6321
18	51.40	51.96	2199.94	2179.50	0.6153
19	50.42	50.98	2199.38	2178.51	0.6596
20	51.13	51.93	2199.65	2177.56	0.6029
21	49.74	49.98	2198.35	2174.99	0.8627
22	48.67	48.65	2199.79	2177.04	0.8498
23	49.13	49.67	2199.80	2176.98	0.8942
24	49.19	49.79	2199.04	2176.76	0.8491
25	51.11	51.95	2199.33	2176.83	0.9390

Bar Num	Area 1 GA Ideal Model P=9327 kgp	Area 2 GA Real Model P=9551 kgp	Stress Area 2 Real Model (kgp/cm^2)	Stress Area 2 Ideal Model (kgp/cm^2)
26	55.3	55.21	1999.85	1971.81
27	58.54	62.10	1999.77	1972.09
28	60.0	59.68	2000.02	1977.21
29	41.12	44.58	1999.28	1976.78
30	67.3	65.69	1999.82	1982.28
31	81.84	83.19	1999.21	1976.67
32	70.1	72.41	1998.46	1936.64
33	57.44	58.40	1999.87	1944.16
34	40.6	44.57	1998.99	1835.00
35	25.8	31.87	1997.68	1710.05
36	73.98	78.93	1999.61	1948.29
37	63.4	67.09	1999.23	1882.44
38	50.2	54.37	1998.84	1844.33
39	37.2	42.49	1999.29	1759.91
40	23.1	30.38	1988.77	1552.85
41	75.5	79.52	1999.68	1960.35
42	64.1	67.57	1999.76	1883.05
43	51.2	54.73	1998.10	1864.79
44	38.6	42.95	1998.82	1796.33
45	23.4	30.43	1998.30	1578.02
46	73.7	80.69	1999.58	1939.92
47	63.0	68.20	1999.16	1838.92
48	49.8	55.47	1999.42	1801.80
49	39.2	43.40	1998.19	1799.99
50	23.0	30.48	1988.96	1513.50
51	69.5	69.55	1951.18	1928.40
52	70.2	73.82	1999.26	1976.64
53	71.9	71.35	1978.05	1959.82
54	52.6	54.12	1985.42	1966.90
55	76.5	72.81	1999.07	1983.70

Results Test Cases X & Y (Discussion)

Comparing the Discrete Solution versus the Continous Solution approximated to the Discrete

Test Case Y

Ideal Model

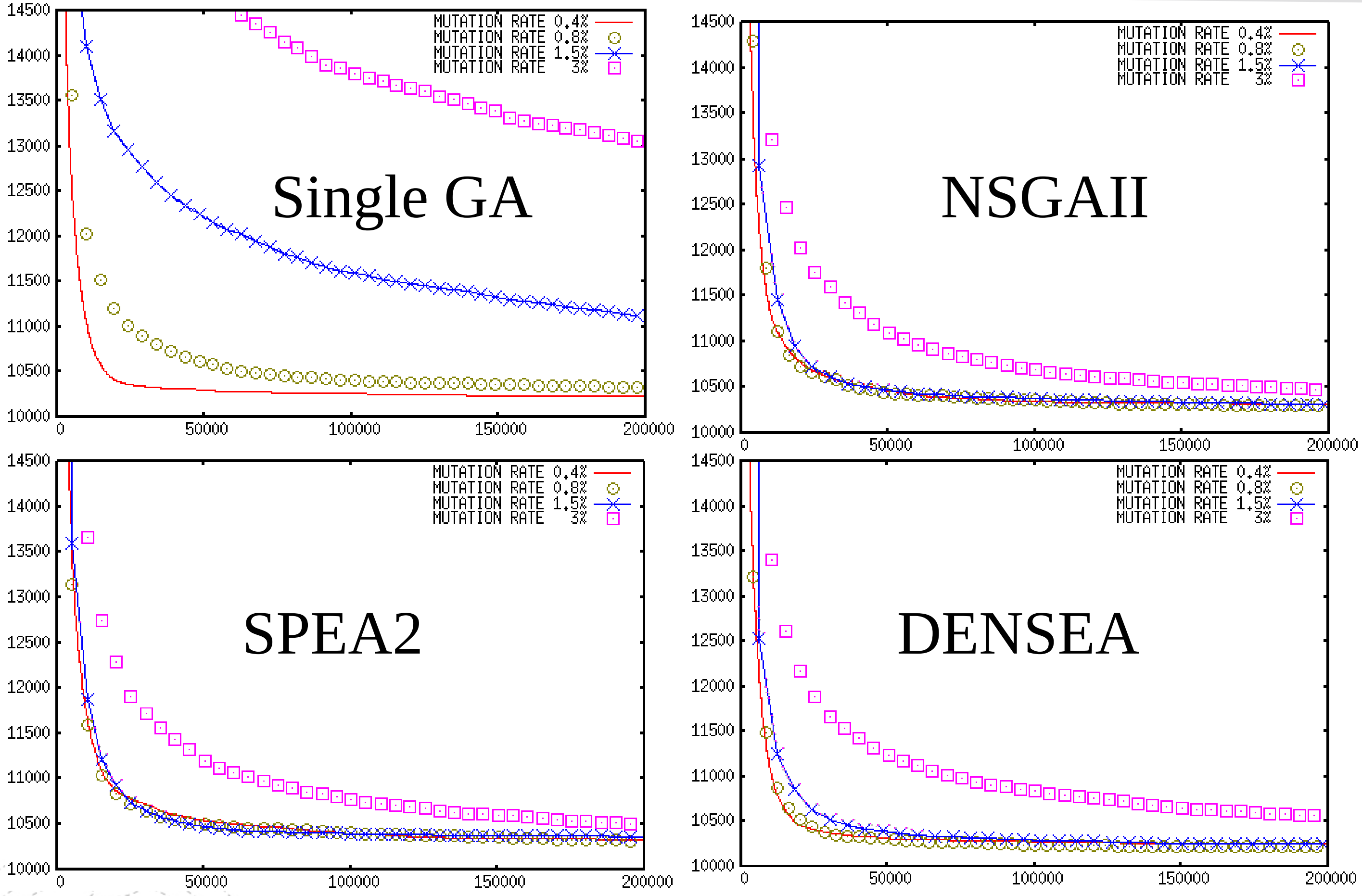
Real Model

Contin. Area GA (P=**9327** kgp.)	Approx. Cross-Section (P=**10031** kgp.)	Discrete Cross-Section (P=**9852** kgp.)
Contin. Area GA (P=**9551.75** kgp.)	Approx. Cross-Section (P=**10343.78** kgp.)	Discrete Cross-Section (P=**10127.3** kgp.)

This results emphazise the need of a *discrete optimizer*

D. Greiner, Emperador, Winter; *Computer Methods in Applied Mechanics and Engineering, Elsevier, 2004*

Helper Objectives - Multiobjectivization

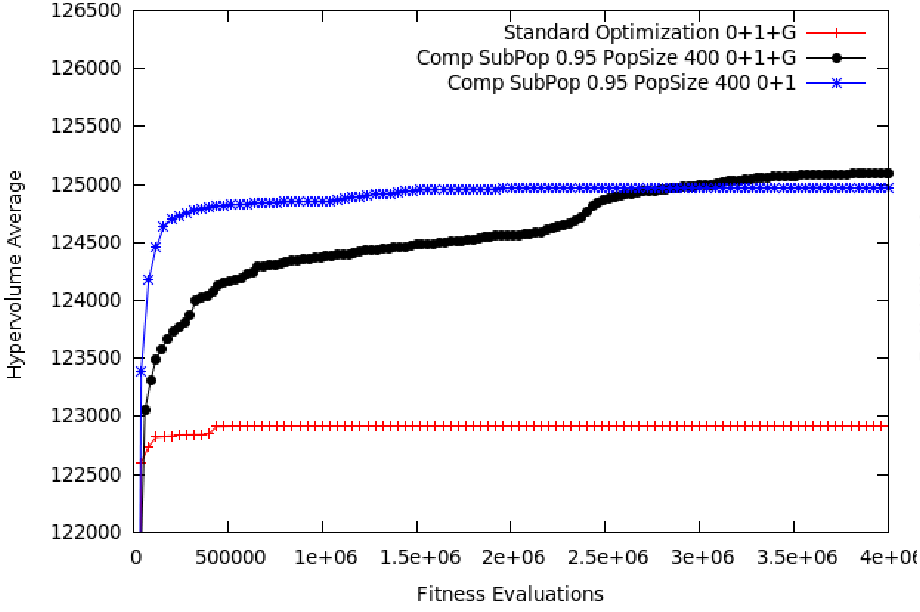


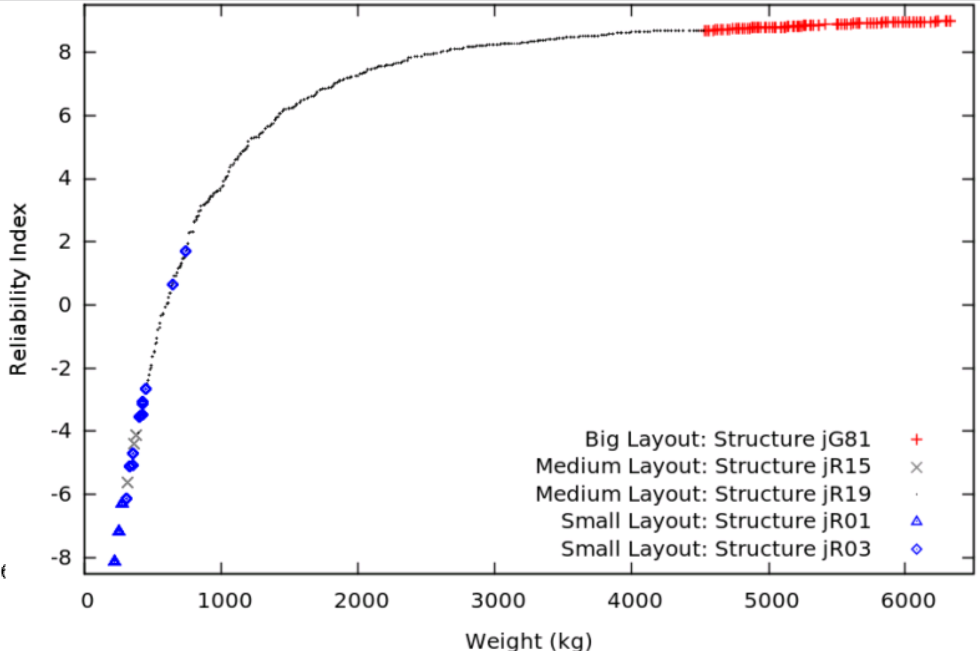
Gray Coding

Test Case Y

D. Greiner, Emperador, Galvan, Winter; *LNCS, Evolutionary Multicriterion Optimization, Springer, 2007*

CoSAR for Reliability Structural Optimization

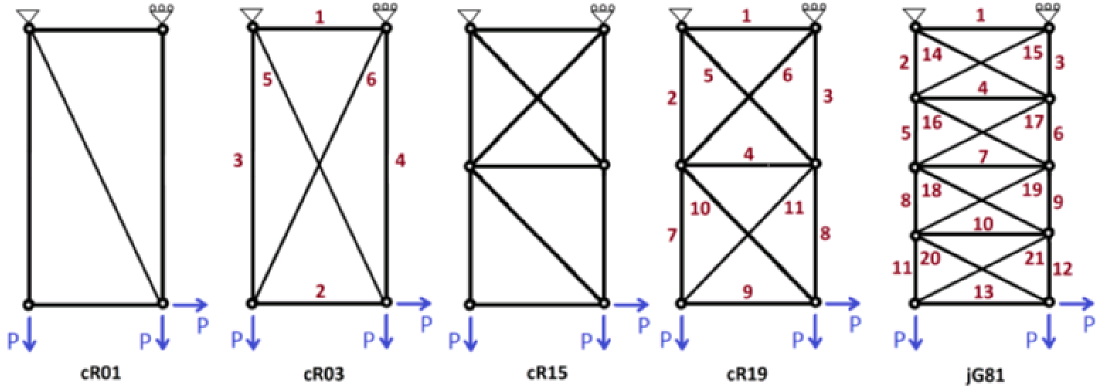




D. Greiner, P. Hajela,

Structural and Multidisciplinary

Optimization, Springer, 2012



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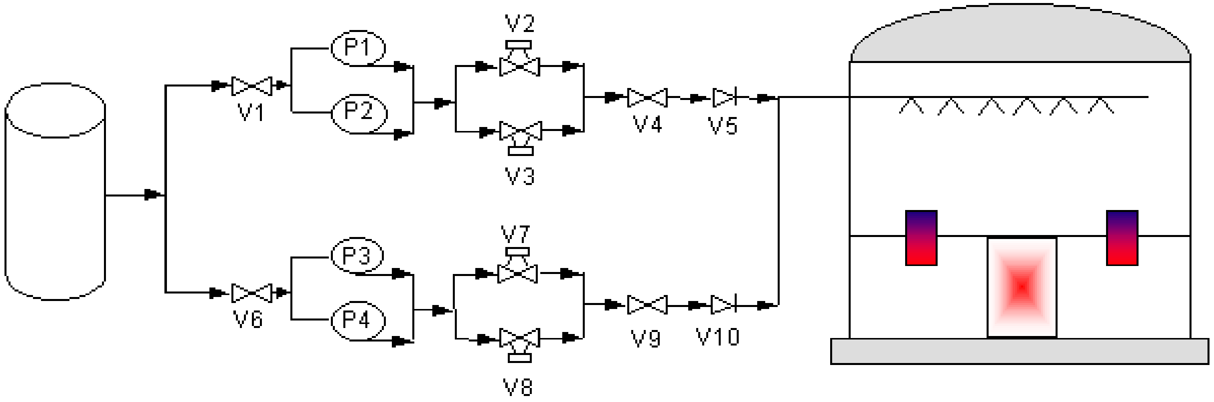
Safety Systems Optimum Design

Objective: To design optimally a safety / protection system, minimizing simultaneously:

1. The material cost of the facilities (obtained by summing the individual costs of the installed elements)
2. The unavailability of the system (depending on the unavailability of the elements considered in the design).

Both are opposing functions, where the diminishing of one , implies the increase of the other. It is required a multicriteria optimization.

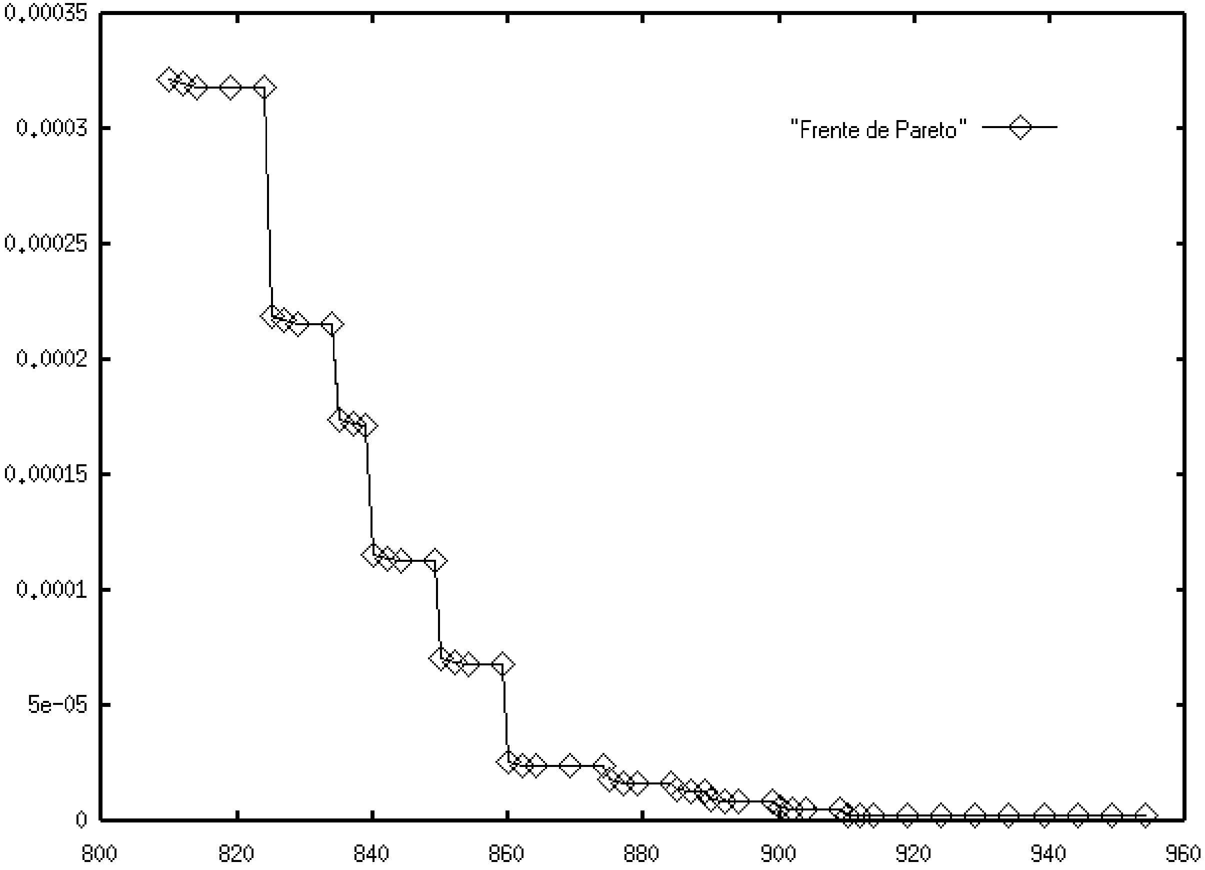
Safety Systems Optimum Design



	V1, V4	V2, V3	V5	P1,P2
Model 1	P=2.9E-3 C=50	P=3.0E-3 C=65	P=5.0E-4 C=37	P=3.5E-3 C=90
Model 2	P=8.7E-3 C=35	P=1.0E-3 C=70	P=6.0E-4 C=35	P=3.8E-3 C=85
Model 3	P=4.0E-4 C=60			

D. Greiner, Galvan, Winter; *LNCS,*
Evolutionary Multicriterion Optimization, Springer, 2003

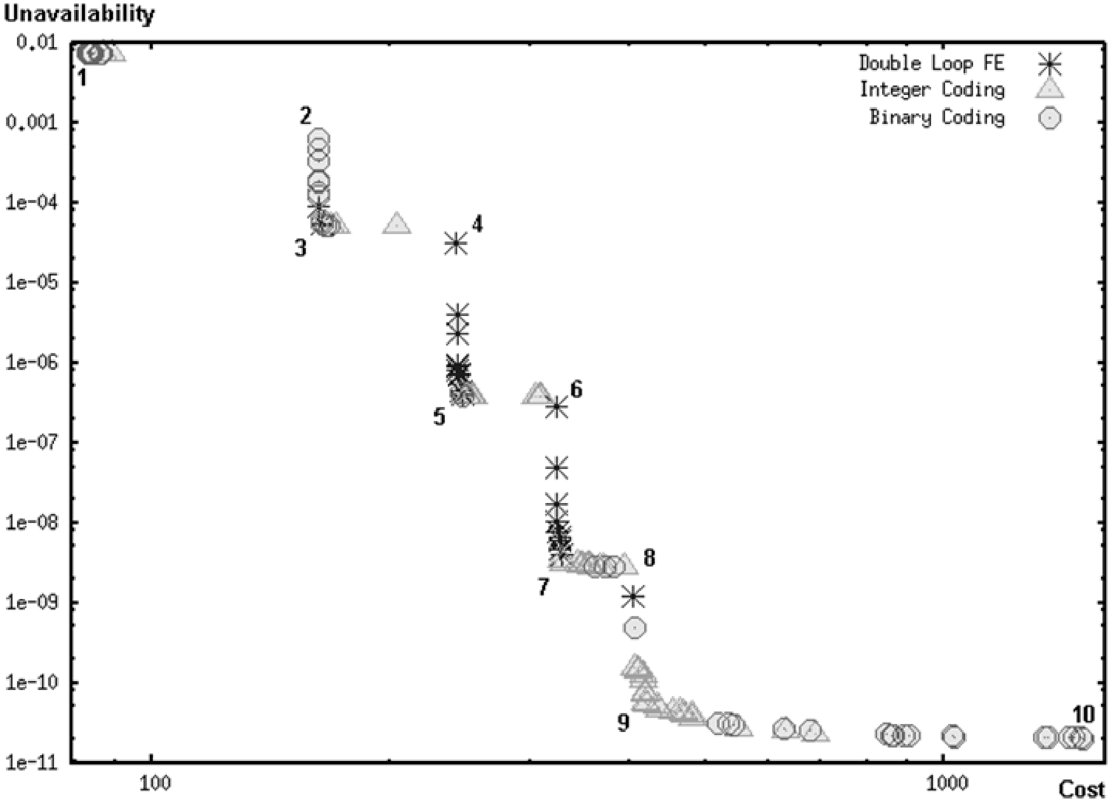
Safety Systems Optimum Design

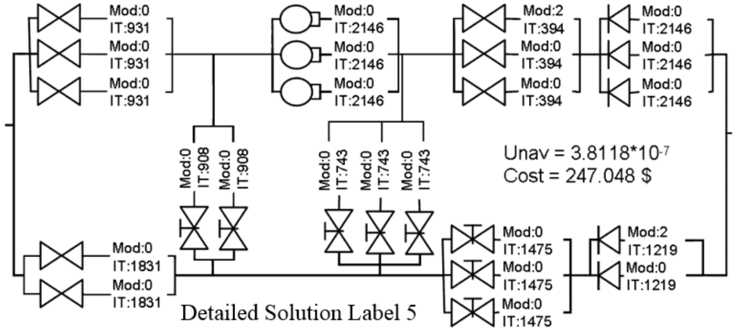


D. Greiner, Galvan, Winter; *LNCS,*
Evolutionary Multicriterion Optimization, Springer, 2003

Pareto Front

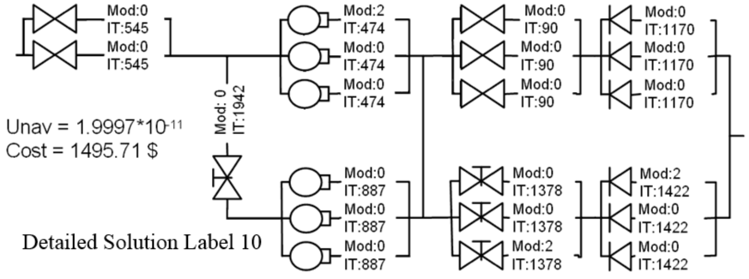
Safety Systems Optimum Design





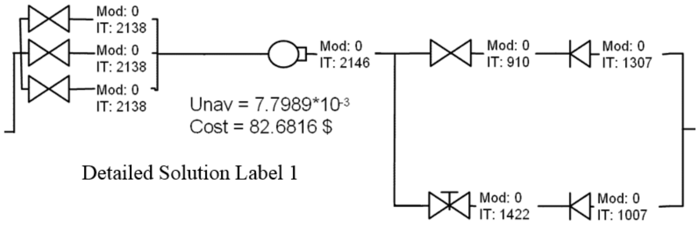
Unav = $3.8118*10^{-7}$
Cost = 247.048 $

Detailed Solution Label 5



Unav = $1.9997*10^{-11}$
Cost = 1495.71 $

Detailed Solution Label 10



Unav = $7.7989*10^{-3}$
Cost = 82.6816 $

Detailed Solution Label 1

B. Galvan, G. Winter, D. Greiner, D. Salazar

New Evolutionary Methodologies for Integrated Safety System Design and Maintenance Optimization, Computational Intelligence in Reliability Engineering (SCI) 39, 151–190 (2007) © Springer-Verlag

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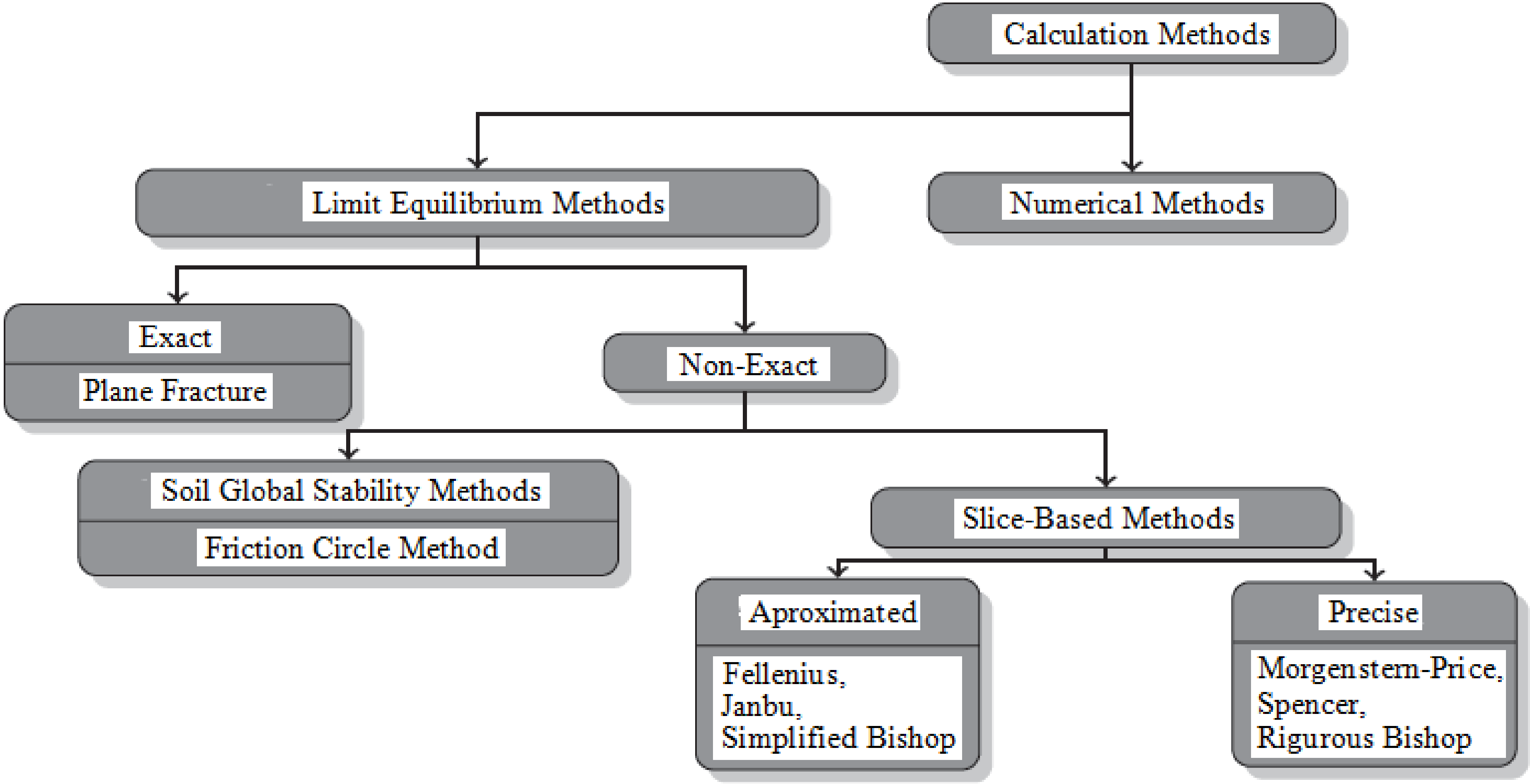
Slope Stability Analysis



Calculation Methods

Limit Equilibrium Methods

Numerical Methods

Exact

Plane Fracture

Non-Exact

Soil Global Stability Methods

Friction Circle Method

Slice-Based Methods

Aproximated

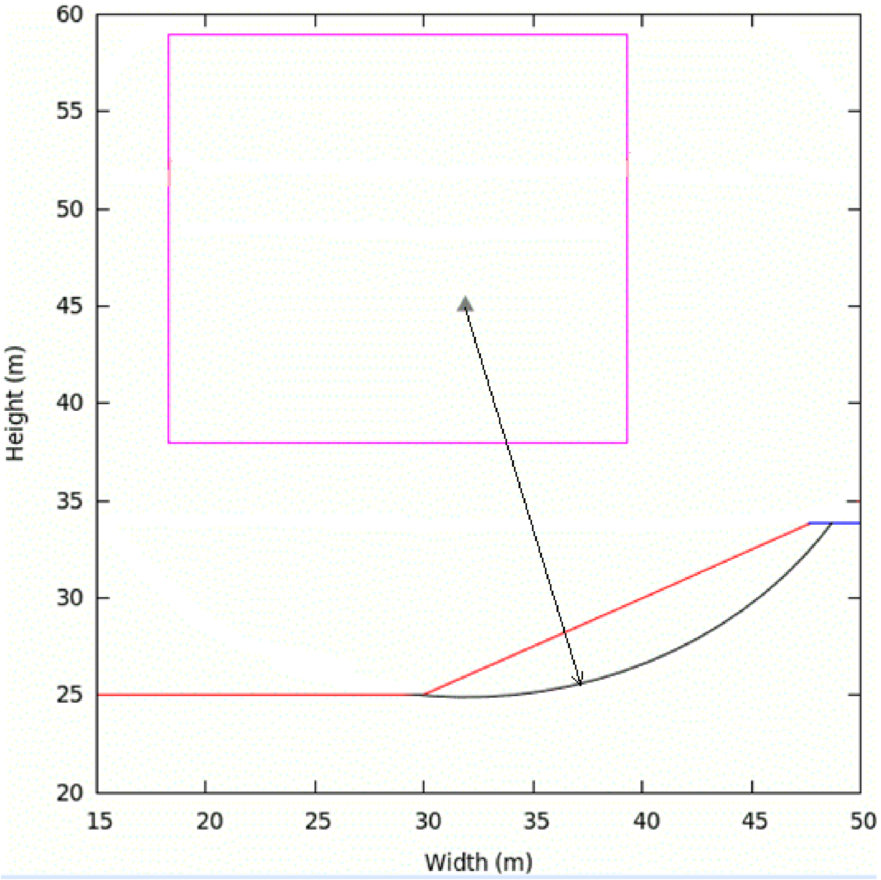
Fellenius,
Janbu,
Simplified Bishop

Precise

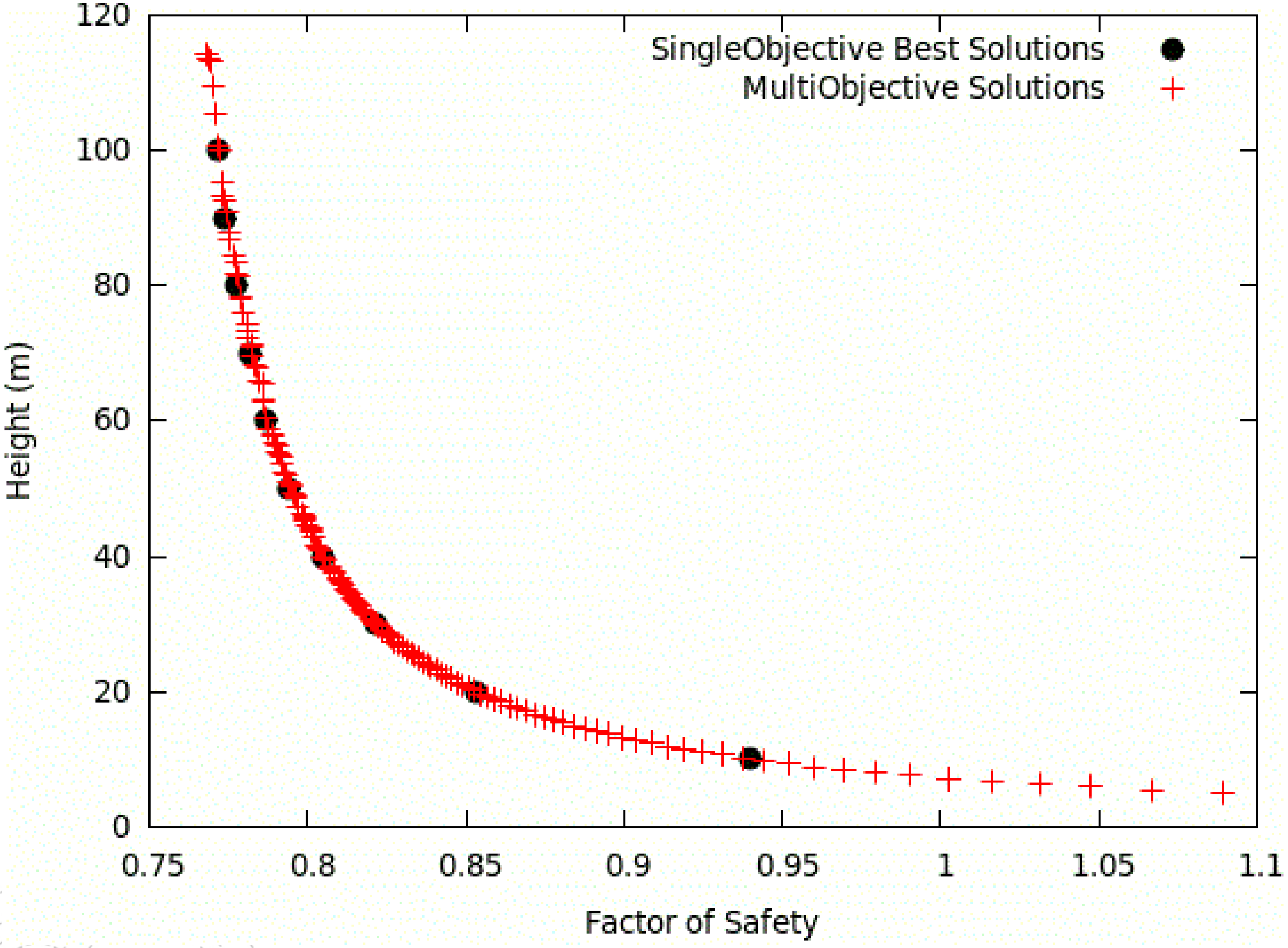
Morgenstern-Price,
Spencer,
Rigurous Bishop

Slope Stability Analysis

- The slip surface with lower factor of safety determines the critical failure surface of a slope; e.g. in case of a circular surface:



Slope Stability Analysis - Results



NSGA2 optimization;

Each red cross represents an optimum design corresponding to the minimum factor of safety for this height

(associated with its critical surface - circle center coordinates & radius value).

D. Greiner, F. Chirino, B. Galván, JM. Emperador, G. Winter; ECCOMAS 2012

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Noise Barrier Design Optimization

The selected objective is to minimize the fitness function (*FF)* as:

$$FF = \sum_{j}^{NFreq} \left(IL_j - IL_j^R \right)^2$$

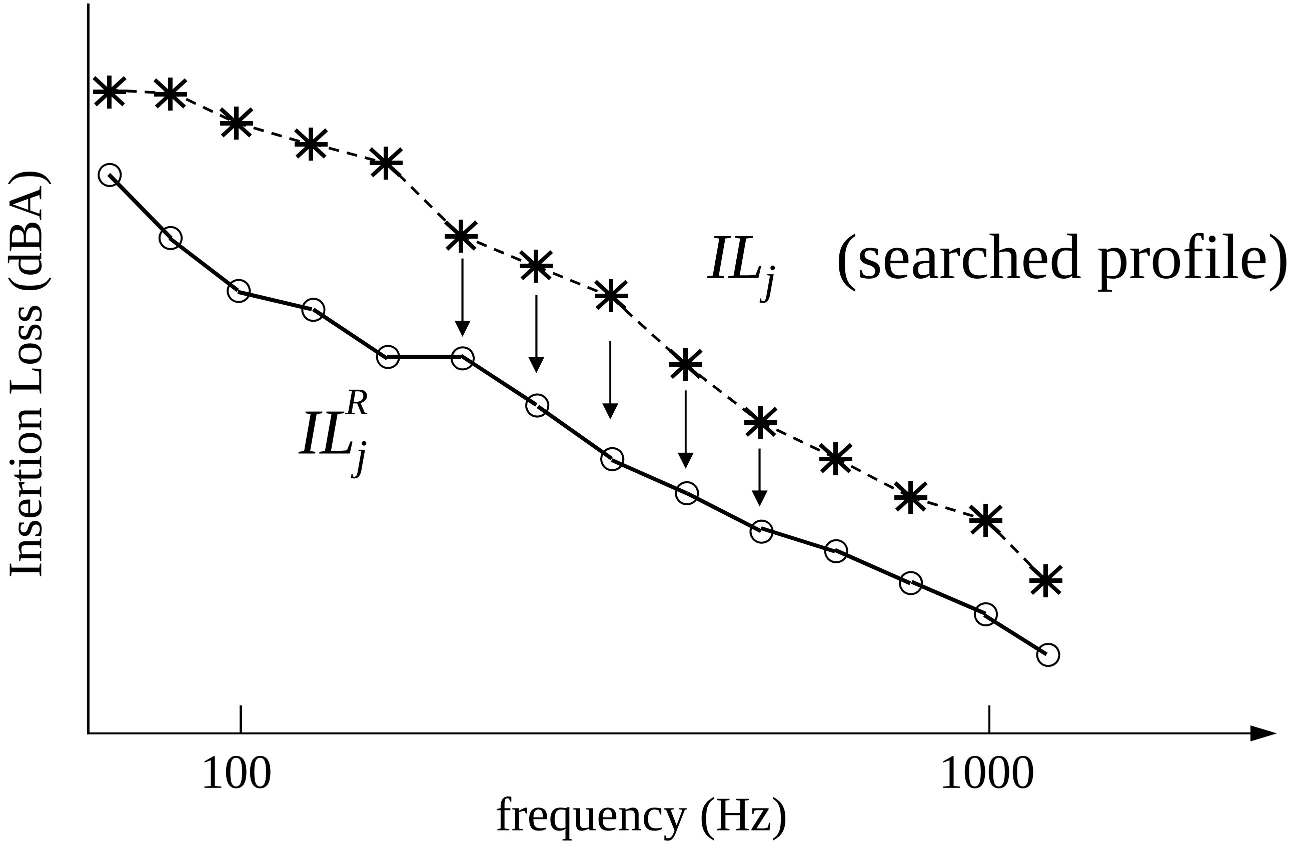
D. Greiner, JJ. Aznarez, O. Maeso, G. Winter,

Advances in Engineering Software, 2010

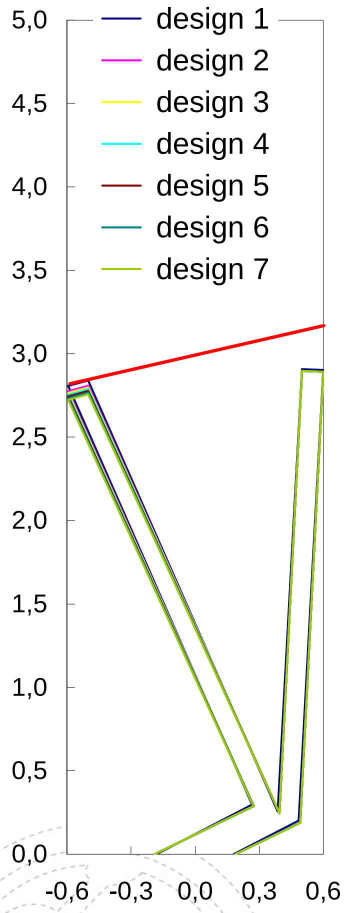
IL values of the candidate solution; IL^R values taken as reference; for each (*j*) octave band or one-third octave band centre frequency.

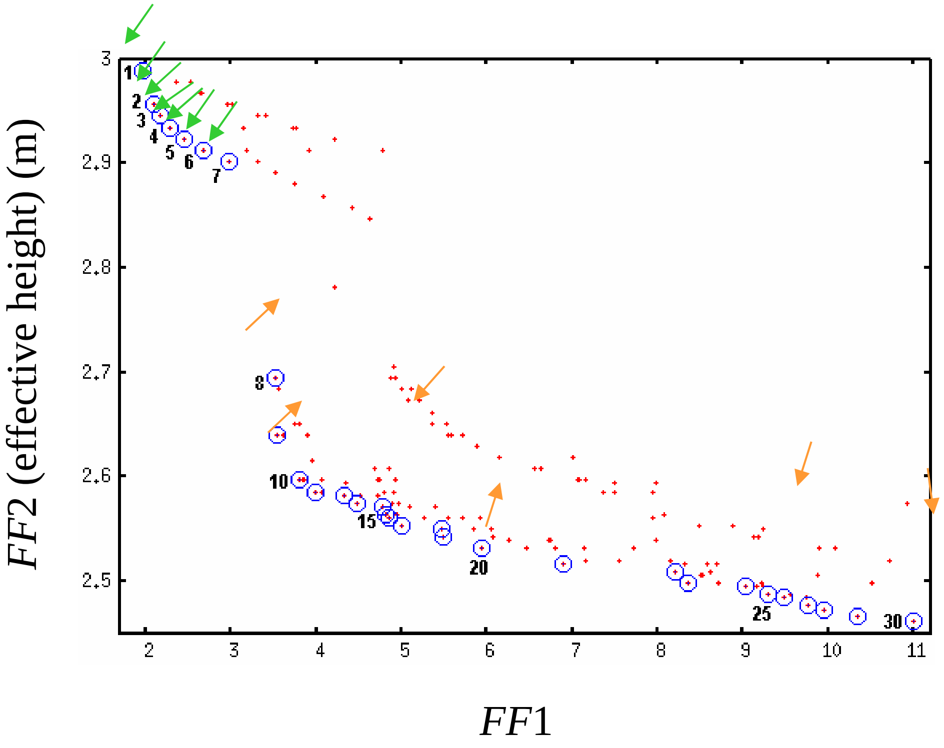
Shape optimum design considering various frequencies is more accurate with respect to the real sound propagation problem, and also allows surpassing the possible problems associated with one single frequency optimization, that could guide to false IL values due to frequencies near to spurious eigenfrequencies associated to the BEM evaluation.

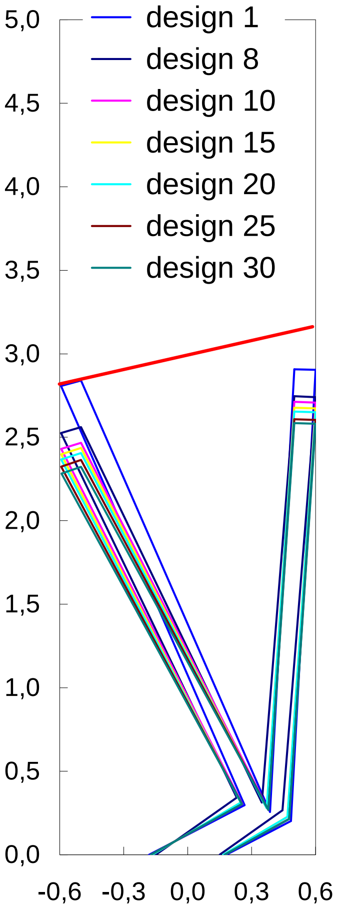
Noise Barrier Design Optimization



Noise Barrier Multi-objective optimization Y-Shape



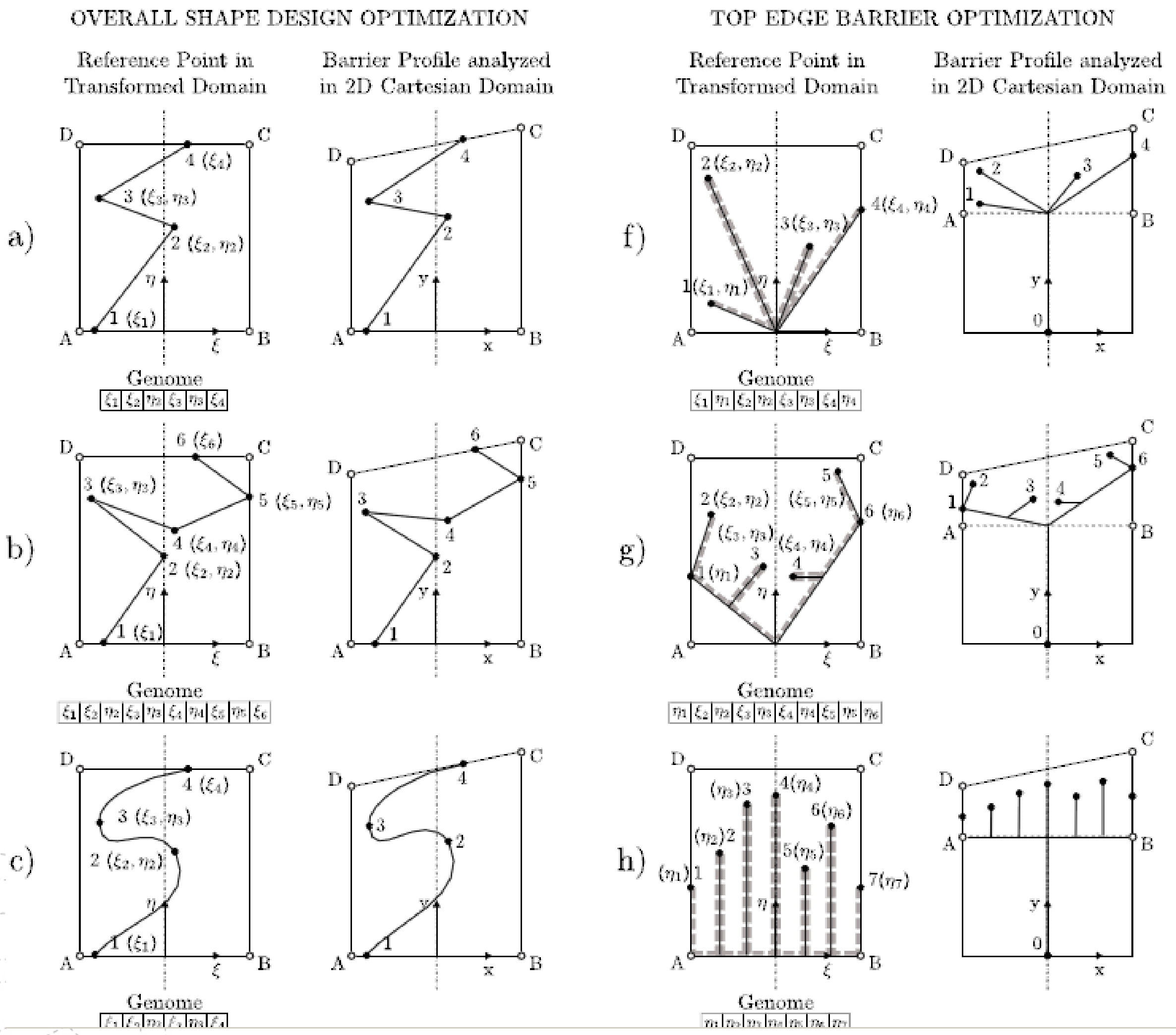




D. Greiner, JJ. Aznarez, O. Maeso, G. Winter,

Advances in Engineering Software, 2010

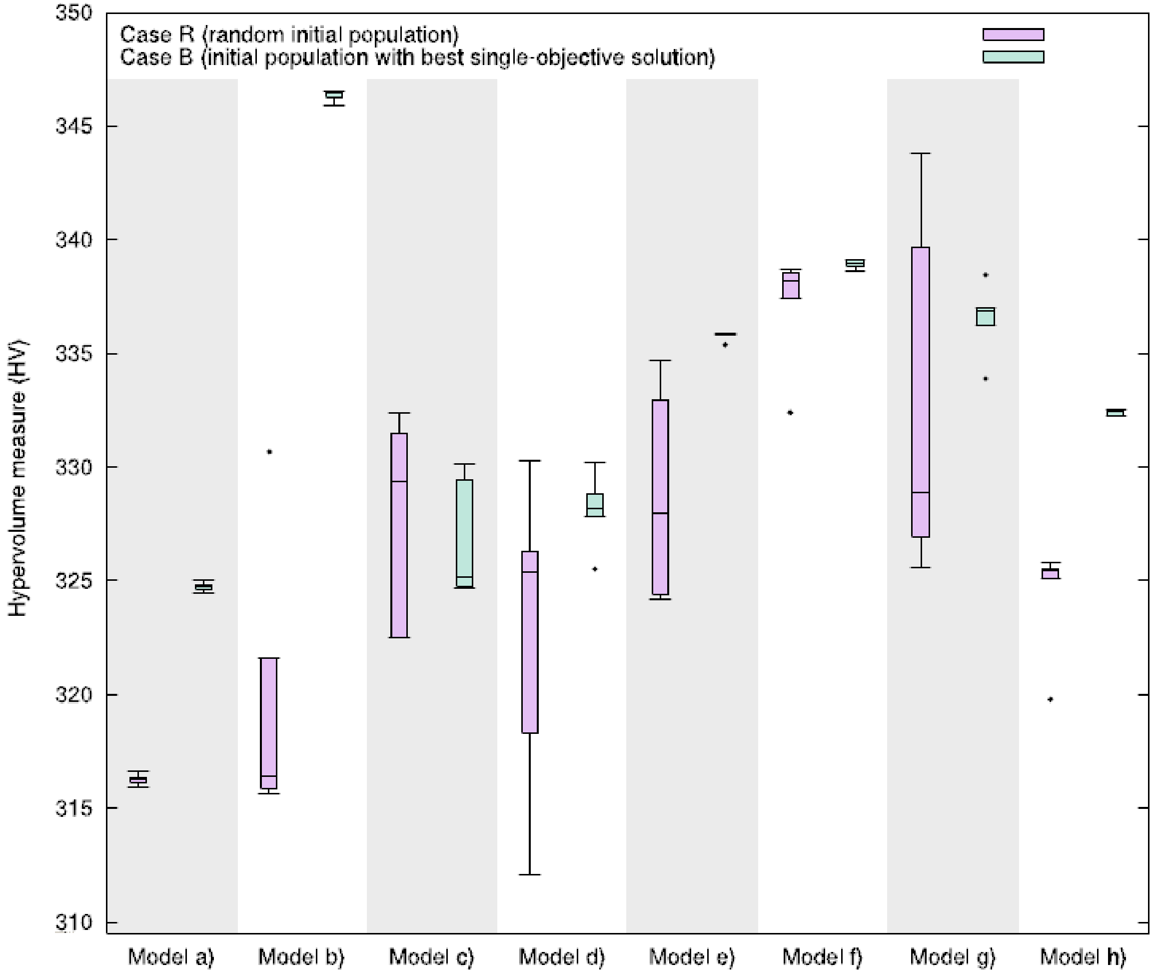
General Noise Barrier Shapes



Journal Sound Vibration, Elsevier, 2015

R. Toledo, J.J. Aznarez, O. Maeso, D. Greiner

General Noise Barrier Shapes



Applied Mathematical Modeling, Elsevier, 2017

R. Toledo, J.J. Aznarez, D. Greiner, O. Maeso

Multiobjective Evolutionary Algorithms for Engineering Optimum Design

David Greiner, J.M. Emperador, B. Galván, F. Chirino,
R. Toledo, J.J. Aznarez, O. Maeso, Gabriel Winter

Institute of Intelligent Systems & Numerical Applications in Engineering -SIANI
Universidad de Las Palmas de Gran Canaria, Spain

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Thank you for your attention !!

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