

Computer-Assisted Proofs in Partial Differential Equations

XXI “Antonio Valle” Prize Lecture

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In the last 50 years computing power has experienced an enormous development: every two years the number of transistors has doubled since the 1970’s. However, even nowadays when we can perform computations at the speeds of the order of Petaflops we can not avoid the following questions, still fundamental in the rigorous analysis of the output of a computer program:

- Q1: Is a computer result influenced by the way the individual operations are done?
- Q2: Does the environment (operating system, computer architecture, compiler, rounding modes, . . .) have any impact on the result?

Sadly, the answer to these questions is Yes. Even the simplest algorithms need a careful analysis: only two operations suffice to give different results if executed in different order or with different rounding methods.

Fortunately, there is a way to give meaningful and true results. In this talk I will explain the basics of interval analysis and how

it can be used to prove theorems in PDE, highlighting recent results in the field.

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