Longest-Edge N-Section for Mesh Refinement, Properties and Open Problems.

José Pablo Suárez*, Agustín Trujillo†, Ángel Plaza* and Tania Moreno‡

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We present a review of most important achievements in the study of geometric properties for the Longest-Edge (LE) mesh refinement.

A complete discussion is made for general N-section. Specially for n=2 and for two and three-dimensional triangulations, we have a series of results that focuses on the generation of a finite number of different triangular and tetrahedral shapes, [1].

Probably the most important result, for the Finite Element Method, is the non-degeneracy condition. This means that the simplices which appear during the refinement process must not degenerate, i.e., the interior (planar, solid) angles of all the elements must be bounded uniformly away from zero.

For example, it is well known that if $n = 2$ and in dimension 2, then LE bisection algorithm produces only a finite number of different triangular shapes, and that this method does not degenerate. A different result but concerning n=3 (Longest-Edge Trisection) confirms that fact at least, the smallest angles do not drop from the initial minimum angle divided by a constant approximately equal to 6,7052.

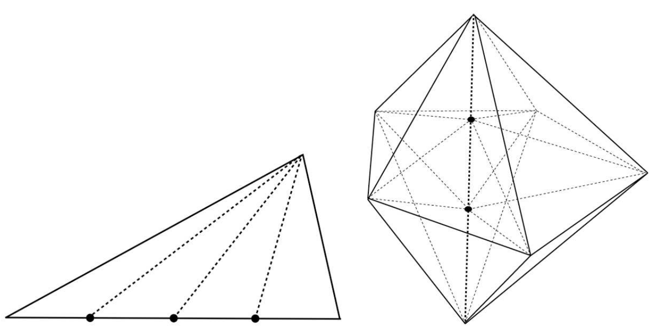

Figure 1: LE 4-section of a triangle with three inserted points on its longest edge (left). The trisection ($n = 3$) of tetrahedra around the longest edge with two points marked (right).

In the case $n \geqslant 4$ and dimension 2, we have that the iterative application of the classical LE n-section algorithm to any given triangle always generates a sequence of sub-triangles whose minimum angles tend to zero, [3].

Most of the proofs in the literature are limited to the two-dimensional case, and not so many mathematical results for LE n-sections are obtained in three and higher dimensions. This leads to an uncertain situation, for example with the three-dimensional case of Longest-Edge subdivision of tetrahedra, where results for minimum solid angles are still open. However, empirical evidence seems to show that this method produces regular families of tetrahedral partitions, [2, 4, 5] although this is not confirmed as no formal proof has not been found. We provide in this work ongoing new results, specially for the three-dimensional case, a revision of previous known results, and remarks of some open problems.

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*Division of Mathematics, Graphics and Computation (MAGiC), IUMA, Information and Communication Systems (SPAIN). Email1: josepablo.suarez@ulpgc.es Email2: aplaza@dmat.ulpgc.es
†Imaging Technology Center (CTIM), University of Las Palmas de Gran Canaria (SPAIN). Email: agustin.trujillo@ulpgc.es
‡Faculty of Mathematics and Informatics. University of Holguin (CUBA).Email: tmorenog@uho.edu.cu