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Staggered schemes for compressible flows

Raphaèle Herbin*

In recent years, a class of schemes has been developed for the numerical resolution of fluid flow in the context of nuclear safety. These flows are complex in nature in that they may include high and low Mach zones. Staggered schemes involve discrete velocity unknowns at the faces and scalar unknowns at the center of the cells and are naturally stable for incompressible regimes; they are thus well qualified for the discretisation of such flows. The well known Marker And Cell (MAC) scheme is an example of such a staggered scheme for rectangular meshes.

The presentation will deal with this class of schemes for the compressible Euler equations. The main features of the schemes, besides the use of a staggered mesh, is the discretisation of the (non-conservative) internal energy balance rather than the total energy, and an unwinding of the fluxes relative to the material speed, therefore natively ensuring the positivity of the density and the internal energy. Time discretization can be explicit or performed by a pressure correction technique. In both cases, a correction term is added to the discrete internal energy equation which allows to recover the correct shock speeds. Moreover, the schemes are proven to be consistent in the Lax sense: indeed, provided compactness assumptions, the sequence of approximate solutions can be shown to converge to a weak solution of the Euler equations. A discrete entropy inequality is obtained when the discretization of the mass balance and internal energy equations is implicit and the fluxes upwind. For explicit discretizations, the current theoretical results require a constraint on the time step that is stronger than the CFL constraint or a specific stabilization term.

It is worthile noticing that for a constant density, the semi-implicit scheme degenerates to a standard algorithm for the incompressible regime. In fact, in the case of the compressible barotropic Navier-Stokes equations, it has been shown that for a fixed mesh, the numerical solution of the compressible scheme converges to the numerical solution of the incompressible scheme as the Mach number tends to zero.

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*Département de mathématiques, Aix-Marseille Université (AMU), Marseille(FRANCE).