

Variational methods for non-variational problems

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We propose and describe an alternative perspective for the study of systems of boundary value problems governed by ODEs. It is based on a variational approach that seeks to minimize a certain quadratic error understood as a deviation of paths from being a solution of the corresponding system. We distinguish two situations depending on whether the problem has or has not divergence structure. In the first case, the functional is not a typical integral functional as the ones examined in the Calculus of Variations, and we have to resort to the Palais-Smale condition to show existence of minimizers. In the case of a fully non-linear problem, however, the functional is an integral, local functional of second order. We illustrate the method with some numerical experiments.

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